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Reconstruction of a permeability field with the wavelet multiscale–homotopy method for a nonlinear convection–diffusion equation

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ABSTRACT

This paper deals with the reconstruction of a piecewise constant permeability field for a nonlinear convection–diffusion equation, which arises as the saturation equation in the fractional flow formulation of the two-phase porous media flow equations. This permeability identification problem is solved through the minimization of a cost functional which depends on the discrepancy, in a least-square sense, between some measurements and associated predictions. In order to cope with the local convergence property of the optimizer, the presence of numerous local minima in the cost functional and the large computational cost, a wavelet multiscale–homotopy is developed, implemented, and validated. This method combines the wavelet multiscale inversion idea with the homotopy method. Numerical results illustrate the global convergence, computational efficiency, anti-noise ability of the proposed method.

1. Introduction

This paper investigates the reconstruction of a permeability field $\mathbf{q}$ for the following nonlinear convection–diffusion equation:

$$u_t + \frac{\partial}{\partial x} f(u) + \frac{\partial}{\partial y} g(u) - \nabla \cdot (\mathbf{q} \cdot N(u) \nabla u) = s(\mathbf{x}, t), \quad (\mathbf{x}, t) \in \Omega \times (0, T), \tag{1.1}$$

with the initial condition

$$u(\mathbf{x}, 0) = u_0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \tag{1.2}$$

and the boundary condition

$$u(\mathbf{x}, t) = 0, \quad (\mathbf{x}, t) \in \partial\Omega \times (0, T), \tag{1.3}$$

where f and g are respectively the nonlinear S-shaped flux functions of Buckley–Leverett type in the x- and y-direction, N is the positive nonlinear function, and s is the piecewise smooth source function.

Eq. (1.1) rises as the saturation equation in the fractional flow formulation of the two-phase porous media flow equations (see [1]):

$$\varphi(\mathbf{x}) u_t + \nabla \cdot (f(u)\mathbf{v} + \mathbf{q} \cdot f_g(u) \nabla \eta) - \nabla \cdot (\mathbf{q} \cdot N(u) \nabla u) = s_1(\mathbf{x}, t), \tag{1.4}$$

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$$\nabla \cdot \mathbf{v} = s_2(\mathbf{x}, t), \tag{1.5}$$

$$\mathbf{v} = -\mathbf{q} \cdot \epsilon(\mathbf{x}, u)(\nabla \overline{p} - \phi(u)\nabla \eta), \tag{1.6}$$

$$f_g(u) = (\phi(u) - \phi_0(u)) f(u) \epsilon_0(\mathbf{x}, u), \tag{1.7}$$

where φ is the porosity, u is the saturation of the wetting phase, f is the nonlinear S-shaped fractional flow function, $\mathbf{v}$ is the total Darcy velocity, $\mathbf{q}$ is the permeability, η is the height, N is the nonlinear diffusion function, s_1 and s_2 are the production and injection wells, ϵ is the total mobility of the phases, ϵ_0 is the phase mobility of the non-wetting phase, $\overline{p}$ is the global pressure, ϕ and ϕ_0 are respectively the densities of the wetting and non-wetting phases.

Apart from the time derivative and convection terms, Eq. (1.1) closely resembles Eq. (1.4). In case $\varphi(\mathbf{x}) = 1$, the time derivative terms in Eqs. (1.1) and (1.4) are equal. The convection terms between Eqs. (1.1) and (1.4) are different only because the convection term in Eq. (1.1) has no varying coefficient and no permeability dependence.

Parameter identification problem based on these equations has shown significant potential in management of petroleum reservoirs. It can help people select the type of the recovery method, fluid production and injection rates, and well locations. Generally speaking, to solve this inverse problem is very difficult, the main reasons lie in:

(1) Ill-posedness: The solution does not depend continuously on the measurement data. A small disturbance of the measurement data may cause big change on the solution of the inverse problem.
(2) Nonlinearity: Nonlinear dependence of the measurement data with respect to the permeability field to be reconstructed causes the presence of numerous local minima.
(3) Large computational cost: The direct model is described by the solution of a nonlinear convection–diffusion equation which is computationally demanding to solve, so the computational cost of this inverse problem is often very large.

As a result, the key problem is how to quickly find a stable solution in a wide range. The aim of this paper is to develop a parameter identification method to overcome these difficulties.

There exists a large number of research in the literature on the effective methods for the reconstruction of $\mathbf{q}$ within the linear elliptic equation

$$-\nabla \cdot (\mathbf{q} \cdot \nabla u) = s(\mathbf{x}) \tag{1.8}$$

(see [2–7] and references therein), and the linear parabolic equation

$$u_t - \nabla \cdot (\mathbf{q} \cdot \nabla u) = s(\mathbf{x}, t), \tag{1.9}$$

such as [8–13] to name a few. On the reconstruction of $\mathbf{q}$ within the nonlinear parabolic equation

$$u_t - \nabla \cdot (\mathbf{q} \cdot N(u, \nabla u) \nabla u) = s(\mathbf{x}, t), \tag{1.10}$$

few studies [14] have been done.

Both Eqs. (1.8) and (1.9) can be considered as the descriptions of one-phase flow processes in porous media where $\mathbf{q}$ is the permeability. Eq. (1.8) corresponds to modeling of single-phase porous-media flow with constant fluid density and viscosity, and Eq. (1.9) corresponds to modeling of slightly compressible single-phase flow with constant compressibility, viscosity and porosity. Eq. (1.10) has interesting mathematical properties related to two-phase porous media flow, although it is not used in two-phase flow simulations. For the reconstruction of a permeability field in the practical application, the interested readers are invited to refer to, e.g., [15–18].

Multiscale method was first studied to solve the forward problem, for example, the two-phase immiscible flow simulations in heterogeneous porous media [19,20], and then was extended to the inverse problem [21]. As a specific form of multiscale methods, wavelet multiscale method has recently emerged in the field of inversion. Liu [22] showed that the wavelet multiscale method could be applied to the distributed parameter estimation of the one-dimensional elliptic equation. Fu and Han [23] and Fu et al. [24] applied the wavelet multiscale method to the reconstruction of a velocity field for a two-dimensional acoustic wave equation. Zhang et al. [25] and He and Han [26] developed the wavelet multiscale method for the reconstruction of a porosity field in a fluid-saturated porous media. Ding et al. [27] considered the wavelet multiscale method to solve the reconstruction of a conductivity field for Maxwell equations. For electrical capacitance tomography [28] and diffuse optical tomography [29], wavelet multiscale method was shown to be very effective. It is shown in these papers that the wavelet multiscale method can speed up convergence, enhance stability of inversion and avoid impact of local minima to reach the global minimum.

Homotopy method has become a powerful tool in finding solutions of various nonlinear problems such as zeros and fixed points of mappings and so on. A distinct advantage of this method is that the algorithm generated by it exhibits the global convergence under certain weak assumptions [30]. It is interesting to apply the homotopy method to the inverse problems. Successful applications of this method include the inverse problem of an elliptical equation [7], the inverse problem of a two-dimensional acoustic wave equation [31,32], the PEM inversion of ARMAX models [33], and the well-log constraint waveform inversion [34]. All these works showed the effectiveness of homotopy method on the inverse problems.

As a matter of fact, for the reconstruction of a permeability field, only the homotopy method may fail to get the global minimum due to the presence of numerous local minima in the cost functional; when the wavelet multiscale method is used singly, the global minimum at the longest scale cannot be obtained if the cost functional may also not be convex. Therefore, an obvious way is to combine these two methods. The whole inversion process is conducted by the wavelet multiscale method, and