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Hybrid Euler–Taylor matrix method for solving of generalized linear Fredholm integro-differential difference equations

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ABSTRACT

The main purpose of this paper is to present a numerical method to solve the linear Fredholm integro-differential difference equations with constant argument under initial-boundary conditions. The proposed method is based on the Euler polynomials and collocation points and reduces the integro-differential difference equation to a system of algebraic equations. For the given method, we develop the error analysis related with residual function. Also, we present illustrative examples to demonstrate the validity and applicability of the technique.

1. Introduction

In this study, we consider the high-order generalized linear Fredholm integro-differential-difference equations with constant arguments (advanced, neutral or delayed) and variable coefficients

$$\sum_{k=0}^{m_1}\sum_{j=0}^{n_1} P_{kj}(x)y^{(k)}(x+\tau_{kj}) = \sum_{r=0}^{m_2}\sum_{s=0}^{n_2}\int_0^b K_{rs}(x,t)y^{(r)}(t+\lambda_{rs})dt + g(x) \tag{1}$$

with the mixed conditions

$$\sum_{k=0}^{m_1-1} (a_{ik}y^{(k)}(0) + b_{ik}y^{(k)}(b)) = \eta_i, \quad i = 0, 1, \ldots, m_1-1, m_1 \geq m_2 \tag{2}$$

where $P_{kj}(x)$, $K_{rs}(x,t)$ and $g(x)$ are known functions defined on the interval $0 \leq x, t \leq b < \infty$; $\tau_{kj}, \lambda_{rs}, \eta_i, a_{ik}$ and b_{ik} are appropriate constants; $y(x)$ is an unknown function to be determined.

The equation defined by (1) is a combination of differential, difference and Fredholm integral equations. This is an important branch of modern mathematics and arises frequently in many applied areas which include engineering, mechanic, physics, chemistry, astronomy, biology, economics, elasticity, plasticity, oscillation theory, etc. [1–9] . In recent years, to solve the mentioned equations, several numerical methods were used such as the successive approximations, Adomian decomposition, Haar wavelet, Block-Pulse, Monte-Carlo, Tau and Walsh series methods [10–13].

Additionally, since the beginning of 1994, Taylor, Chebyshev, Laguerre, Hermite, Bernstein and Bessel methods based on matrix method have been used by Sezer et al. [14–23] to solve linear differential, difference, integral and Fredholm integro-differential-difference equations. Our purpose in this study is to develop a new matrix method, which is based on the Euler basis

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set $\{E_0(x), E_1(x), E_2(x), \ldots, E_n(x), \ldots\}$ [24] and collocation points, to obtain the approximate solution of the problem (1) and -(2) in the Euler polynomial form

$$y(x) \cong y_N(x) = \sum_{n=0}^{N} a_n E_n(x), \quad 0 \le x \le b < \infty, \tag{3}$$

where the Euler basis set is defined by $\{E_0(x), E_1(x), E_2(x), \ldots, E_n(x), \ldots\}$, a_n $(n = 0, 1, 2, \ldots, N)$ are the unknown coefficients to be determined.

The rest of this paper is organized as follows. The fundamental matrix relations with related to the Euler polynomials and their derivatives are presented in Section 2. The new Euler matrix method based on collocation points is described in Section 3 and Section 4. In Section 5, the error analysis technique related to residual function is developed for the present method. To support our results, in Section 6, we present the results of numerical experiments. Section 7 concludes this study with a brief summary.

2. Euler polynomials

Euler numbers and polynomials are very useful in classical analysis and numerical mathematics. Their basic properties can briefly be summarized as follows [24–30].

Euler polynomials $E_n(x)$ are defined by the generating function

$$\frac{2e^{xt}}{e^t + 1} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!}, \; |t| < \pi. \tag{4}$$

Euler numbers ε_n can be obtained by the generating function

$$\frac{2}{e^t + e^{-t}} = \sum_{n=0}^{\infty} \frac{\varepsilon_n}{n!} t^n \tag{5}$$

and the connection between Euler numbers (5) and Euler polynomials (4) is given by

$$E_n(1/2) = 2^{-n}\varepsilon_n, n = 0, 1, 2, \ldots \tag{6}$$

Euler polynomials are strictly connected with Bernoulli ones, and are used in the Taylor expansion in a neighborhood of the origin of trigonometric and hyperbolic secant functions. Recursive computation of Euler polynomials can be obtained by using the following formula;

$$E_n(x) + \sum_{k=0}^{n} \binom{n}{k} E_k(x) = 2x^n, n = 1, 2, \ldots \tag{7}$$

Also, Euler polynomials $E_n(x)$ can be defined as polynomials of degree $n \ge 0$ satisfying the conditions

$$E'_m(x) = mE_{m-1}(x), m \ge 1, \tag{8}$$

$$E_m(x + 1) + E_m(x) = 2x^m, m \ge 1. \tag{9}$$

By using (6) , (7) or (9), the first Euler numbers and Euler polynomials, respectively, are given by

$$\varepsilon_0 = 1, \varepsilon_1 = 0, \varepsilon_2 = -1, \varepsilon_3 = 0, \varepsilon_4 = 5, \varepsilon_5 = 0, \varepsilon_6 = -61, \varepsilon_7 = 0, \ldots$$

and

$$E_0(x) = 1, E_1(x) = x - \frac{1}{2}, E_2(x) = x^2 - x$$

$$E_3(x) = x^3 - \frac{3}{2}x^2 + \frac{1}{4}, E_4(x) = x^4 - 2x^3 + x, \ldots$$

3. Matrix relations for Euler and Taylor polynomials

Let us consider the integro-differential-difference Eq. (1) and find the matrix forms of each term in the equation. Firstly, we can convert the desired solution $y(x)$ defined by the truncated Euler series (3) of Eq. (1) to the matrix form as, for $n = 0, 1, 2, \ldots, N$,

$$y(x) \cong y_N(x) = \mathbf{E}(x)\mathbf{A}, \tag{10}$$

where

$$\mathbf{E}(x) = [E_0(x) \; E_1(x) \ldots \; E_N(x)]$$