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A mathematical model for the customer dynamics based on marketing policy

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ABSTRACT

We consider a compartmental model to study the evolution of the number of regular customers and referral customers in some corporation. Transitions between compartments are modeled by parameters depending on the social network and the marketing policy of the corporation. We obtain some results on the asymptotic number of regular customers and referral customers in several particular scenarios. Additionally we present some simulation that illustrates the behavior of the model and discuss its applicability.

1. Introduction

In marketing literature it has been successively referred the importance of calculating the value of a customer. In fact, such indicative value enables firms to select those customers that can add profit and consequently constitutes an important information to segment the market and efficiently allocate marketing policy resources [7,8,13].

The objective of this work is to establish and study a compartmental model, mathematically translated into a system of ordinary differential equations, for the evolution of the number of customers of some firm, assuming that the customers are divided in two subgroups corresponding to different profitabilities.

Until recently, the value of a customer for a company was based on the present value of future profits generated by a customer over the full course of their dealings with a particular company, this is the customer life-time-value (CLV) [9]. However, other authors refer to the importance of including not only the present and future revenue from the customer purchases, but also the value of the potential to influence other customers under incentives on behalf of the company (customer referral value) or by own initiative (customer influencer value) [8]. Customer influencing behaviors consists of the intrinsic behaviors motivating the customer to persuade and influence other customers without there being any type of reward on behalf of the company and thus designated the customer influencer value (CIV). In turn, the patterns of customer recommendation are related to the acquisition of new customers due to company initiatives that reward recommendations made to other customers, and thereby establishing the customer referral value (CRV). According to Kumar et al. [9], these components are mutually interwoven. Thus, CLV positively correlates with CRV (although only up to a certain point and in an inverted U-shaped relational curve, which means customers reporting average CLV are those most interested in company referral programs) and CLV is positively related with CIV (with an inverted U-shaped relationship in effect between these two concepts). Much of the literature has focused on the customer

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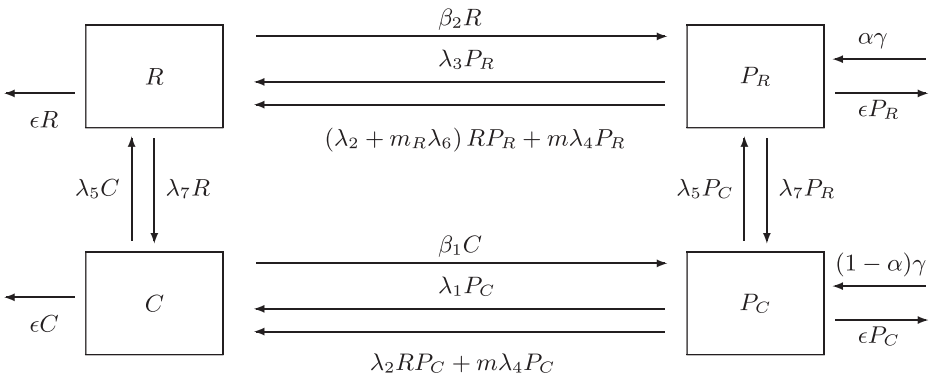

Fig. 1. The compartmental model.

referral value through the influence customers might have on the formation of other customers attitudes ([4]) in the purchasing making decision [2] and in the reduction of other customers perceived risks [5], but little is known about how this processes occur. Since customer referral value and influencer value might have a great impact for companies, these latter try to identify the most influential customers [6].

A number of studies allow us to think that the customers of a firm can be classified into several groups according to their influential role over other potential buyers. In imperfect competitive markets information is not purely transparent; some persons are more able than others of influencing people to become a customer of that firm. It is also acceptable to assume that knowing the referrals among each firms customers and quantifying their influence constitutes an important asset for the firm competitive advantage, although all customers are important, referrals would be more valuable.

We mostly agree with Marti and Zenou [12] when they state that physics/applied mathematics are capable of reproducing many real networks but never reach to explain why they emerge; the economists are very precise to explain why they emerge but their approach does a poor job in matching real world networks. That is why some game theorists are now improving models which take networks as given entities and study the impact of their structure on individuals outcomes.

Based on the network theory some models have been tested to study the way influential customers can influence other consumers. For instance Kiss and Bichler [6] tested real network models, simulated networks and diffusion models to predict influence between customers based on their position within the network. However, as the authors mention this analysis not always is possible if we do not know or do not have information regarding the customer social network. Therefore, other models are needed to try to explain these processes.

In this work we propose a model suitable to describe the dynamics of the number of customers of a given firm. This model is given by a system of ordinary differential equations whose variables correspond to groups of customers and potential customers divided according to their profile and whose parameters reflect the structure of the underlying social network and the marketing policy of the firm. We intend to understand the flows between these groups and its consequences on the raise of customers of the firm. We also want to highlight the usefulness of these models in helping firms deciding their marketing policy.

Specifically, the main objectives of our study is threefold: we intend to obtain theoretical results concerning the long term behavior of the number of customers in various scenarios, we want to present some simulation aimed at illustrating the possibilities of application of our model and, finally, we want to discuss the benefits and limitations of this type of analysis.

As referred, we will consider a compartmental model. As far as we are aware, this is the first time such type of mathematical model is considered in the context of marketing research. We believe that this type of model can be fruitfully explored in this context. This believe is based on the fact that compartmental models have proved to be an important tool not only in the natural sciences, particularly in mathematical epidemiology [3] and in population biology [15,17], but also, with increasing notoriety in recent years, in the context of economy and other social sciences [1,11,14,16].

We consider a continuous compartmental model with four compartments, represented by the graph in Fig. 1 and governed by an autonomous system of four ordinary differential equations.

Measuring time in years, we consider the following (pairwise disjoint) compartments: $R(t)$, the referral customers in time t, $C(t)$, the regular customers in time t, $P_R(t)$, the potential referral customers in time t and $P_C(t)$, the potential regular customers in time t. To model transitions between compartments we consider the following parameters: λ_1, the natural transition rate between P_C and C, given by the number of potential regular customers that become regular customers without external influence per year over the number of potential customers (by "without external influence" we mean without being influenced by marketing campaigns or referral customers); λ_2, the referral pull effect, given by the average number of customers that a single referral brings (with no additional incentive) per year over the number of potential customers; λ_3, the natural P_R to R transition rate, corresponding to the number of potential referral customers that become referral customers without external influence per year over the number of potential referral customers; m, the undifferentiated marketing costs, corresponding to marketing costs associated to undifferentiated marketing campaigns per year; λ_4, the pull effect due to undifferentiated marketing, corresponding to the quotient of the outcome of undifferentiated marketing campaigns per year by the number of potential customers (by