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A collocation method based on Bernstein polynomials to solve nonlinear Fredholm–Volterra integro-differential equations

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ABSTRACT

In this study, a collocation method that based on Bernstein polynomials is presented for nonlinear Fredholm–Volterra integro-differential equations (NFVIDEs). By means of the collocation method and the matrix operations, the problem is reduced into a system of the nonlinear algebraic equations. The approximate solutions are obtained by solving this nonlinear system. Error analysis is presented for the Bernstein series solutions of the nonlinear Fredholm–Volterra integro-differential equations. Several examples are given to illustrate the efficiency and implementation of the proposed method for solving the NFVIDEs. Comparisons are made to confirm the reliability of the method. Also error analysis is applied for the numerical examples.

1. Introduction

Many scientific phenomena in science and engineering can be modeled by nonlinear integro-differential equations. Nonlinear integro-differential equations are usually difficult to solve analytically. So, it is important that the approximate solutions of the nonlinear integro-differential equations can be computed by numerical methods. In recent years, several numerical methods for solving linear and nonlinear integro-differential equations have been presented by many authors. For example, some linear and nonlinear integro-differential equations have been solved using the Taylor expansion approach [1–3], the Tau method [4–6], the Chebyshev method [7], the Taylor method [8,9], the homotopy perturbation method [10,11], the Taylor polynomial method [12], the Shannon wavelets approximation [13], the He's homotopy perturbation method [14], the He's variational iteration method [15], the direct method [16,17], the sinc-collocation method [18,19], the rationalized Haar functions [20], the homotopy analysis method [21], the Cas wavelet method [22], the Bessel collocation method [23,24], the Chebyshev matrix method [25], the multi-parametric homotopy approach [26] and the B-spline collocation method [27].

In this study, we present a Bernstein series method for the solutions of the mth order nonlinear Fredholm–Volterra integro-differential equation

$$\sum_{k=0}^{m}\sum_{r=0}^{n} P_{k,r}(x)y^{r}(x)y^{(k)}(x) = g(x) + \lambda_1\int_a^b K_F(x,t)[y(t)]^p dt + \lambda_2\int_a^x K_V(x,t)[y(t)]^q dt, \qquad 0 \le a \le x,t \le b \tag{1}$$

with the mixed conditions

$$\sum_{k=0}^{m-1}[a_{jk}y^{(k)}(a) + b_{jk}y^{(k)}(b)] = \gamma_j, \quad j = 0,1,\ldots,m-1 \tag{2}$$

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where $y^0(x) = 1$ and $y^{(0)}(x) = y(x)$ are unknown functions, p and q are nonnegative integers, $\lambda_1, \lambda_2, a_{jk}, b_{jk}$ and γ_j are constants. Moreover, $P_{k,r}(x)$, $g(x)$, $K_F(x,t)$ and $K_V(x,t)$ are continuous functions on interval $a \le x, t \le b$.

We note that some researchers have considered the various studies regarding existence of solutions of integral and integro-differential equations in [28–33]

This paper is organized as follows: some properties of Bernstein polynomials are given in Section 2. In Section 3, we summarize the method. The Bernstein series method is applied for nonlinear Fedholm–Volterra integro-differential equations in Section 4. In Section 5, we give the error analysis for the method. We present five numerical examples to clarify the method in Section 6. Section 7 concludes the article with a brief summary.

2. Some properties of Bernstein polynomials

The Bernstein basis polynomials of degree n (see, [34], e.g., [35], p. 66) are defined by

$$B_{k,n}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad x \in [0,1].$$

By using the binomial expansion

$$(1-x)^{n-k} = \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} x^i,$$

it can be written as

$$B_{k,n}(x) = \sum_{i=0}^{n-k} (-1)^i \binom{n}{k} \binom{n-k}{i} x^{k+i}, \quad x \in [0,1].$$

Also, the Bernstein basis polynomials of degree n in $[0, R]$ are given by the formula [34]

$$B_{k,n}(x) = \binom{n}{k} \frac{x^k (R-x)^{n-k}}{R^n}. \tag{3}$$

By substituting the binomial expansion

$$(R-x)^{n-k} = \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} R^{n-k-i} x^i$$

here, we have the formula

$$B_{k,n}(x) = \sum_{i=0}^{n-k} (-1)^i \binom{n}{k} \binom{n-k}{i} \frac{x^{k+i}}{R^{k+i}} x \in [0,R].$$

The Bernstein basis polynomials given by Eq. (3) can be written in the matrix form [36–38]

$$\mathbf{B}_n(x) = [B_{0,n}(x)\ B_{1,n}(x)\ \cdots\ B_{n,n}(x)] = \mathbf{X}(x)\mathbf{D}^T \tag{4}$$

where

$$\mathbf{X}(x) = [1 \quad x \quad x^2 \quad \cdots \quad x^n], \quad \mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{bmatrix},$$

and

$$d_{ij} = \begin{cases} \dfrac{(-1)^{j-i}}{R^j} \dbinom{n}{i} \dbinom{n-i}{j-i}, & i \ge j \\ 0, & i < j. \end{cases}$$

3. Description of the Bernstein series method

In this study, by using the Bernstein polynomial approximation [36–38], we obtain an approximate solution of the problem given by Eqs. (1) and (2) in the form

$$y_N(x) = \sum_{k=0}^{N} a_k B_{k,N}(x-c), \quad x \in [0,b]. \tag{5}$$