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Convergence studies on block iterative algorithms for image reconstruction☆

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ABSTRACT

The sequential block-iterative scheme based on Landweber's method is a general iterative one for image reconstruction. In this paper, we give the finite termination convergence for this scheme, which provides an approach to choose relaxation coefficients. Furthermore, sufficient and necessary convergence conditions are also established for the sequential block-iterative scheme case.

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1. Introduction

Large systems of linear equations arise in many areas of scientific computing and engineering applications, e.g., many imaging systems such as computerized tomography (CT) and magnetic resonance imaging (MRI) and so on (see [19–30] and references therein). The image reconstruction problem can be modeled as the following systems of linear equations

$$Ax = b, \tag{1.1}$$

where A is an $M \times N$ nonzero matrix, $x \in K^N$ and $b \in K^M$ denote the desired image and the observed data, respectively. K is a field of real numbers or complex numbers. Iterative methods have the capacity of producing better quality images when measured data contain noises or are not sufficient. Based on the classification in [9], iterative methods can be generally grouped into sequential, simultaneous and block-iterative (sequential or simultaneous) methods. Byrne [5] pointed out that block-iterative (or called as ordered-subset) methods are between the algebraic reconstruction technique (ART, also called Kaczmarz's method, [21]) and the Landweber algorithm [23]. These three iterative methods uses some, only one or all measured data at a time, respectively. ART is regarded as a sequential block-iterative algorithm, while the Landweber algorithm is treated as a simultaneous block-iterative algorithm [9]. When the number of blocks is only one, a sequential block-iterative algorithm is simplified as a simultaneous algorithm. More detailed discussions on sequential and simultaneous structures can also be obtained in, e.g., [9], the review paper [2], and [20].

Starting from a given initial approximation, the above algorithms update the approximations step by step using fixed or variable relaxation parameters, one or a few at a time in a certain order, until the convergence is realized. Censor and Elfving [7] gave

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the convergence of the sequential block-iterative scheme in the cases of no preconditioners and consistence, while Jiang and Wang [20] established a unifying framework for convergence of block-iterative methods. For the Landweber scheme, necessary and sufficient conditions for its convergence have been given and convergence within finite iterations has been proved in [27]. But no necessary and sufficient conditions for convergence of the sequential block-iterative scheme till now. For overdetermined systems of linear equations, Bai and Jin [1] presented a class of column-decomposed relaxation methods and established its convergence theory under suitable conditions. Bai and Liu [3] established new convergence theorems for row-action iteration schemes such as the block Kaczmarz and the Householder–Bauer methods used to solve large linear systems and least-squares problems. By transforming sequences and restarting, Brezinski and Redivo-Zaglia [4] accelerate Kaczmarz's method for solving consistent systems of linear equations. There have been a few papers on how to choose parameters for some particular convergent methods. Censor et al. [8] selected relaxation parameters based on numbers of nonzero elements in column of block and diagonal elements of positive definite diagonal weight matrices for diagonally-relaxed orthogonal projection methods. Relaxation parameters are not changed with iterations in the using of the block iterations with symmetric positive definite weights [13]. Regarding the simultaneous iterative reconstruction technique (SIRT) for computed tomography as a Richardson iteration, the relaxation parameter in [17] was chosen to be $2/(1+\epsilon)$ with ϵ positive but far from one. Also for the SIRT methods, Elfving et al. [14] proposed two specified techniques to control the noise errors. Recently, a MATLAB package for algebraic iterative reconstruction methods was released (see [18]), which contained several strategies to choose the relaxation parameters based on the above choices, and stopping rules based on [12,25]. However, these values of relaxation parameters are still selected empirically. There is no automated method for selecting them [17]. In this paper we try to give a step to that direction. When the roots of some polynomial equations are chosen to be the relaxation parameters, the sequential block-iterative algorithms will be proved to converge in finite steps. We give the necessary and sufficient convergence conditions for sequential block-iterative algorithms in this paper, which includes the case of the Landweber scheme.

Also, problem (1.1) can be described as the convex feasibility problem (CFP), see [9,10] and references therein. In [6,15], finite convergence of iterative projection algorithms for CFP was used to study the convergence for CFP. Here finite convergence means that from a certain iteration index onward the algorithm does not create further changes of the iteration vectors.

In this paper, we will first present the sequential block-iterative algorithm, and its iterative formula based on singular value decomposition (SVD, [16]). Then we will prove that the scheme converges within finite iterations. Finally, sufficient and necessary convergence conditions for the sequential block-iterative algorithm are given for the sequential block-iterative algorithm.

2. The sequential block-iterative algorithm

In this section, a general block-iterative scheme based on Landweber's method [23] is introduced in [20].

We first present simultaneous block-iterative scheme for solving the following system of linear equations

$$x_{n+1} = x_n + \alpha_n P^{-1} A^* W (b - A x_n), \quad n = 0, 1, \ldots, \tag{2.1}$$

where α_n is a relaxation coefficient, preconditioner P^{-1} is a symmetric positive definite (SPD) matrix of order N, weight W is a SPD matrix of order M and A^* denotes complex conjugate of matrix A. We use $[n]$ to denote $n(\text{mod } m)+1$, where m is the number of block. Let $B = \{1, \ldots, M\} = \cup_{1\le t\le m} B_t$. Corresponding to the blocks $B_1, B_2, \ldots, B_m$, matrix A is partitioned into $A_1, A_2, \ldots, A_m$ by row. Similarly, $W_1, \ldots, W_m$ and $b_1, \ldots, b_m$ are the corresponding blocks of W and b, respectively. So the sequential block-iterative scheme for solving the system of linear Eq. (1.1) can be defined as follows [20]:

$$x_{n+1} = x_n + \alpha_n P^{-1} A^*_{[n]} W_{[n]} (b_{[n]} - A_{[n]} x_n), \quad n = 0, 1, \ldots. \tag{2.2}$$

To write the scheme (2.2) into the following inner–outer iteration scheme (e.g. see [26]), so we get the following sequential block-iterative scheme

$$\begin{cases} x_0 \quad \text{arbitrary}, \\ x_{n,1} = x_n, \\ x_{n,k+1} = x_{n,k} + \alpha_{n,k} P^{-1} A_k^* W_k (b_k - A_k x_{n,k}), \\ x_{n+1} = x_{n,m+1}, \\ k = 1, \ldots, m, \quad n = 0, 1, \ldots. \end{cases} \tag{2.3}$$

If multiplying $P^{\frac{1}{2}}$ from left on scheme (2.3), we obtain

$$P^{\frac{1}{2}} x_{n,k+1} = P^{\frac{1}{2}} x_{n,k} + \alpha_{n,k} P^{-\frac{1}{2}} A_k^* W_k^{\frac{1}{2}} (W_k^{\frac{1}{2}} b_k - W_k^{\frac{1}{2}} A_k P^{-\frac{1}{2}} P^{\frac{1}{2}} x_{n,k}).$$

Set

$$y_{n,k} = P^{\frac{1}{2}} x_{n,k}, \quad G_k = W_k^{\frac{1}{2}} A_k P^{-\frac{1}{2}}, \quad \bar{b}_k = W_k^{\frac{1}{2}} b_k, \tag{2.4}$$

then scheme (2.3) can be rewritten as

$$y_{n,k+1} = y_{n,k} + \alpha_{n,k} G_k^* (\bar{b}_k - G_k y_{n,k}), \quad k = 1, \ldots, m, \quad n = 0, 1, \ldots, \tag{2.5}$$

or

$$y_{n,k+1} = (I - \alpha_{n,k} G_k^* G_k) y_{n,k} + \alpha_{n,k} G_k^* \bar{b}_k, \quad k = 1, \ldots, m, \quad n = 0, 1, \ldots.$$

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