Applied Mathematics and Computation 273 (2016) 637–644

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Application of Fibonacci collocation method for solving Volterra–Fredholm integral equations

Farshid Mirzaee*, Seyede Fatemeh Hoseini

Faculty of Mathematical Sciences and Statistics, Malayer University, P.O. Box 65719-95863, Malayer, Iran

ARTICLE INFO

MSC:
65D30
41A10
45G10

Keywords:
Volterra–Fredholm integral equation
Fibonacci collocation method
Error estimate

ABSTRACT

In this paper, a new matrix method based on Fibonacci polynomials and collocation points is proposed for numerically solving the Volterra–Fredholm integral equations. In fact, the approximate solution of the problem in the truncated Fibonacci series form is obtained by this method. Also, convergence analysis of the proposed method is provided under several mild conditions. The reliability and efficiency of the proposed scheme are demonstrated by some numerical experiments.

1. Introduction

Various types of integral equations appear in many fields of science and engineering. One of them is the Volterra–Fredholm integral equation that arises from parabolic boundary value problems, the mathematical modeling of the spatio-temporal development of an epidemic and various physical and biological models [1]. These equations are usually difficult to solve analytically, so it is required to obtain the approximate solutions. Many numerical methods have been studied for approximating the solution of Volterra–Fredholm integral equations, such as Taylor polynomials method [2–5], homotopy perturbation method [6], Legendre wavelets method [7], interpolation method [8], Chebyshev polynomials method [9], rationalized Haar functions method [10] and spectral method [11]. Consider the Volterra–Fredholm integral equations of the form

$$q(s)g(s) + r(s)g(x(s)) = y(s) + \lambda_1 \int_a^{x(s)} k_1(s,t)g(t)dt + \lambda_2 \int_a^b k_2(s,t)g(x(t))dt, \quad (1)$$

where the functions $y : [a, b] \to R,\ k_i : [a, b] \times [a, b] \to R\ (i = 1, 2)$ and x: $[a, b] \to [a, \infty)$ are the known functions and g: $[a, b] \to R$ is the unknown function and a, b and λ_i $(i = 1, 2)$ are constants such that $\lambda_1^2 + \lambda_2^2 \neq 0$. When $x(s)$ is first-order polynomial, the Eq. (1) is functional integral equation with proportional delay.

In this paper, we concern with a collocation method in combination with Fibonacci polynomials method to the numerical solution of the Volterra–Fredholm integral equations of the form (1). It is well known that the collocation-type methods provide highly-accurate approximations for the solutions of linear and nonlinear operator equations in function spaces which is proved that these solutions are sufficiently smooth [12–14]. The proposed method consists of reducing the calculation of the solution of this equation to a set of equations by expanding the candidate function as a Fibonacci function with unknown coefficient as

$$g(s) \simeq \sum_{n=1}^{N+1} g_n F_n(s), \quad 0 \le a \le s \le b, \quad (2)$$

* Corresponding author. Tel.: +98 81 32355466; fax: +98 81 32355466.
E-mail addresses: f.mirzaee@malayeru.ac.ir, f.mirzaee@iust.ac.ir (F. Mirzaee), fatemeh.hoseini@stu.malayeru.ac.ir (S.F. Hoseini).

http://dx.doi.org/10.1016/j.amc.2015.10.035
0096-3003/

where $g_n,\ n = 1, 2, \ldots, N+1$ is the unknown Fibonacci coefficients, N is any arbitrary positive integer and $F_n(s), n = 1, 2, \ldots, N+1$ is the nth Fibonacci polynomial defined by

$$F_{n+1}(s) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i} s^{n-2i}, \quad n \geq 0,$$

where $\lfloor \frac{n}{2} \rfloor$ denotes the greatest integer in $\frac{n}{2}$. Note that for $s = k \in N$ we obtain the elements of the k-Fibonacci sequences.

This paper is structured as follows. In Section 2, we will describe a successful numerical approach which is used for making up the solution. Then, in Section 3, convergence analysis of the presented method is discussed. In Section 4, we report our numerical finding and demonstrate the accuracy of the proposed scheme by considering numerical examples. The conclusions are discussed in the final section.

2. Description of the method

A function $g(s)$, square integrable on $[a, b]$, may be expressed in terms of the Fibonacci basis as (2). In practice, only the first-$(N+1)$-term of Fibonacci polynomials are considered. In the same way, we can write

$$g(x(s)) \simeq \sum_{n=1}^{N+1} g_n F_n(x(s)), \quad 0 \leq a \leq s \leq b. \tag{3}$$

Let us rewrite (1) in the form

$$R(s) = y(s) + \lambda_1 V(s) + \lambda_2 U(s), \tag{4}$$

where

$$R(s) = q(s)g(s) + r(s)g(x(s)),$$
$$V(s) = \int_a^{x(s)} k_1(s,t)g(t)dt,$$

and

$$U(s) = \int_a^b k_2(s,t)g(x(t))dt.$$

Assume that

$$s_m = a + \frac{b-a}{N}(m-1), \qquad m = 1, 2, \ldots, N+1, \tag{5}$$

where the values s_m are spread out over the interval $[a, b]$ and satisfy

$$a = s_1 < s_2 < \cdots < s_{N+1} = b.$$

By substituting these collocation points into (4), we have

$$R(s_m) = y(s_m) + \lambda_1 V(s_m) + \lambda_2 U(s_m). \tag{6}$$

Using (2) and (3), we get

$$R(s_m) \simeq q(s_m) \sum_{n=1}^{N+1} g_n F_n(s_m) + r(s_m) \sum_{n=1}^{N+1} g_n F_n(x(s_m)),$$
$$V(s_m) \simeq \sum_{n=1}^{N+1} g_n \int_a^{x(s_m)} k_1(s_m,t)F_n(t)dt,$$

and

$$U(s_m) \simeq \sum_{n=1}^{N+1} g_n \int_a^b k_2(s_m,t)F_n(x(t))dt.$$

So from above equations, we can rewrite (6) as follows

$$\sum_{n=1}^{N+1} \left[q(s_m)F_n(s_m) + r(s_m)F_n(x(s_m)) - \lambda_1 \int_a^{x(s_m)} k_1(s_m,t)F_n(t)dt - \lambda_2 \int_a^b k_2(s_m,t)F_n(x(t))dt \right] g_n = y(s_m).$$

For simplicity, the above formula can be represented as follows

$$\sum_{n=1}^{N+1} \left[\mathcal{R}(s_m) - \lambda_1 \mathcal{V}(s_m) - \lambda_2 \mathcal{U}(s_m) \right] g_n = y(s_m), \qquad m = 1, 2, \ldots, N+1, \tag{7}$$