Applied Mathematics and Computation 273 (2016) 718–725

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stability analysis of discrete-time switched linear systems with unstable subsystems☆

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ARTICLE INFO

Keywords:
Switched systems
Unstable subsystems
Discrete-time
Exponential stability
Average dwell time

ABSTRACT

The problem of stability for discrete-time switched linear systems (DSLSs) with unstable subsystems is investigated. Unlike most existing results that require each switching mode of the system to be asymptotically stable, this paper considers the case that each subsystem may be unstable. First, under certain hypotheses, a necessary condition of stability for DSLSs is obtained. Second, using the average dwell time (ADT) strategy, some sufficient conditions of exponential stability for switched linear systems are derived under two assumptions. Finally, two examples are presented to show the effectiveness of the proposed approaches.

1. Introduction

Switched systems are a class of simple hybrid systems, which are used for modeling various types of complex processes in nature. In spite of their apparent simplicity, switched systems possess a very complicated dynamical behavior because of the multiple subsystems and various types of switching signals. In the past decade, the stability problem of switched systems has drawn considerable attentions from system and control communities (see [1–14]). According to different time domains, the results on stability for switched systems can be classified into two different parts: continuous-time context and discrete-time context [20–24]. It should be noted that relatively more attentions have been put on continuous-time switched systems [1–9,26–29]. Recently, discrete-time switched linear systems (DSLSs) have also attracted some attentions of scientists and engineers [10–14,16,17,19]. A DSLS consists of a family of discrete-time state-space models and a switching rule orchestrating the switching among the models. In real world, there are many systems that can be modeled by DSLSs, for example, piecewise affine systems, flight control systems, and networks employing TCP.

As well known, the existence of a common quadratic Lyapunov function (CQLF) is only a sufficient condition for the asymptotic stability of switched systems under arbitrary switching [1,2,4,13]. Some attentions have been paid to a less conservative class of Lyapunov functions, called multiple Lyapunov functions (MLFs) [1,3,5,8,25]. Both CQLF and MLF are almost based on the assumptions that the subsystems are asymptotically stable. However, due to the fact that there exist unstable subsystems in practice, carrying out studies on switched systems with unstable subsystems is quite necessary, which is not only theoretically challenging, but also of practical importance. Note that the authors in the literatures [5–7,20,31] investigated the stability of

☆ This work is supported by the National Natural Science Foundation of China (11171197, 10971123, 61203123, 61573069), the Natural Science Foundation of Shanxi Normal University (2R1506 050502080102) and the.

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http://dx.doi.org/10.1016/j.amc.2015.10.039
0096-3003/

switched systems with unstable subsystems. The literature [31] proposed a mixed DT and state-dependent switching strategy for continuous-time switched linear systems with unstable subsystems.

Although dwell time (DT) strategy has played a very important role in stability analysis of switched systems [31,32], ADT switching characterizes a larger class of switching signals than DT switching. As far as the ADT switching is concerned, one of our aims is to identify the minimal ADT for a given switched system, such that the stability or some system performances are satisfied. Recently, a new strategy called mode-dependent ADT/DT switching has been proposed in the literature [8,11,15,32], which is less conservative than general ADT/DT switching because of the new strategy permitting each subsystem of switched systems to have its own ADT/DT. On the other hand, it is worth pointing out that there are also some results obtained by using ADT technique for DSLSs [8,17]. However, most of these results are obtained by using the similar techniques for continuous-time switched systems. But for DSLSs, even if the average dwell time has been larger than the minimal ADT, the switching still cannot occur since the state of a DSLS only depends on sampling times rather than all time instances. In order to solve this contradiction, this paper proposes a new concept of discrete-time switching signal (DTSS) and develops the ADT technique for DTSS. A DTSS is a switching signal that the switching events only occur at the integer sampling instances.

Motivated by the above-mentioned works, in this paper, we consider DSLSs with unstable subsystems sharing a common equilibrium point. Different from most existing results, the developed results of this paper can deal with DSLSs with all subsystems being unstable. Firstly, a necessary condition of stability will be presented for DSLSs. Secondly, by using ADT strategy, some sufficient conditions of stability for switched linear systems with stable and unstable subsystems are proposed. Thirdly, sufficient conditions of stability for DSLSs with all subsystems being unstable are established. Under the given switching signals, the activation time ratio of two sets of unstable subsystems is required to be located between two constants which are computed by using the desired stability degree of a given switched system.

The remainder of the paper is organized as follows. Section 2 gives problem formulation. In Section 3, a necessary condition of stability for switched linear systems with unstable subsystems is presented, and then some sufficient conditions for switched linear systems with unstable subsystems are given in Section 4. The correctness of the obtained results are illustrated by two examples in Section 5. Section 6 is the conclusion.

2. Preliminaries

The notation used in this paper is standard. $\mathbb{R}$ denotes the set of real numbers; $\mathbb{C}$ denotes the set of complex numbers; $\mathbb{Z}_0^+$ (resp., $\mathbb{Z}^+$) denotes the set of non-negative integers (resp., positive integers); $P \geq 0$ (resp., $P > 0$) means that the Hermitian matrix P is nonnegative (resp., positive); $\mathbb{R}^n$ represents the set of $n \times 1$ real column vectors; $\lambda(A)$ and $\rho(A)$ denotes the spectrum and spectral radius of the square matrix A, respectively; $\oplus$ stands for the direct sum; the superscript T stands for the matrix transposition, and I represents the identity matrix with proper dimension; in addition, the matrices are assumed to be compatible for algebraic operations, if their dimensions are not explicitly stated. $dim(M)$ denotes the dimension of a subspace M; for a number set X with finite elements, $max\{X\}$ ($min\{X\}$) means the maximum (minimum) value of X; given two sets A and B, the intersection (union) of A and B is written as $A \cap B$ ($A \cup B$), and $A \subset B$ indicates that A is a subset of B; for $x \in \mathbb{R}^n$, $\|x\|$ and $\|x\|_V$ denote Euclidean vector norm of x and Euclidean vector norm of x with restriction on $V \subset \mathbb{R}^n$, respectively; $\|A\|$ and $\|A\|_V$ denote the spectral norm of A and the spectral norm of A with restriction on $V \subset \mathbb{R}^n$, respectively, where $A \in \mathbb{R}^{n\times n}$, and V is a A invariant subspace.

Consider the following switched system

$$x(t+1) = A_{\sigma(t)}x(t), \quad x(0) = x_0, \quad t \in \mathbb{Z}_0^+ \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and x_0 denote the state and initial state, respectively. Switching signal $\sigma(t) : \mathbb{Z}_0^+ \mapsto \mathcal{I}_\mathcal{N} = \{1, 2, \dots, N\}$ is an arbitrary function where $N > 1$ is the number of subsystems. Let $A_i \in \mathbb{R}^{n\times n}, i \in \mathcal{I}_\mathcal{N}$, be a family of constant matrices describing subsystems. In this paper, we assume that all the subsystems matrices A_i may be unstable.

Some preliminaries are first introduced, which are necessary for developing the main results of this paper.

Definition 2.1. Given an $n \times n$ complex matrix A and a subspace M of $\mathbb{C}^n$, we say that M is A invariant subspace if $AM \subset M$.

Definition 2.2. Consider the i^{th}, $i \in \mathcal{I}_\mathcal{N}$,subsystem of (1): $x(t+1) = A_i x(t)$. Three subspaces can be defined by A_i:

(1) the stable subspace E_i^s, which is spanned by the generalized eigenvectors corresponding to the eigenvalues λ with $|\lambda| < 1$;
(2) the unstable subspace E_i^u, which is spanned by the generalized eigenvectors corresponding to the eigenvalues λ with $|\lambda| > 1$;
(3) the center subspace E_i^c, which is spanned by the generalized eigenvectors corresponding to the eigenvalues λ with $|\lambda| = 1$.

Proposition 2.1. *Let E be an invariant subspace of A_i, then E is an invariant subspace of A_i^s, $s = 1, 2, \dots$.*

Proof. For all $x_0 \in E$, $x_1 = A_i x_0 \in E$. Thus, $x_2 = A_i^2 x_0 = A_i x_1 \in E$. Repeating the above process, $x_s = A_i^s x_0 \in E$ $(s = 1, 2, \dots)$. By the arbitrariness of $x_0 \in E$, E is an invariant subspace of A_i^s $(s = 1, 2, \dots)$. □

All three subspaces are invariant subspaces of the subsystem, i.e., for all $x_0 \in E_i^s(E_i^u, E_i^c)$, $x(t) = A_i^t x_0 \in E_i^s(E_i^u, E_i^c)$. Please see [18,30].

Definition 2.3. A switching signal $\sigma(t)$ is said to be a discrete-time switching signal (DTSS) if switching events of the signal only occur at sampling instances.