Applied Mathematics and Computation 273 (2016) 793–796

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# A note on the convergence order of some recent methods for solving nonlinear equations

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ARTICLE INFO

*Keywords:*
Nonlinear equations
Iterative methods
Newton method
Optimal order

ABSTRACT

In this paper we show that some of the methods presented in Neta et al. (2014) do not possess optimal eighth order of convergence. Such methods are especially those obtained by Hermite interpolation. One of the two methods based on Jarratt's optimal fourth order method possesses the convergence of seventh order whereas the other possesses fourth order. The methods based on King's and Ostrowski's optimal fourth order methods have convergence order six. The theoretical results are also verified through numerical example.

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## 1. Introduction

In recent paper [1], Neta et al. presented various eighth order methods using two-step optimal fourth order methods followed by a step of modified Newton's method. The modification was performed by considering an interpolating polynomial to replace either a function or a derivative. Here, we show that four methods (of eight presented methods) namely, JHID8, JHIF8, HK8 and WLN which are based on Hermite interpolation do not possess optimal eighth order convergence as claimed by the authors. The methods are listed below:

**I. JHID8**: The method is based on Jarratt's optimal fourth order method [2] followed by a Newton-like step in which the derivative is replaced by Hermite interpolating polynomial and is given by

$$\begin{cases} y_n = x_n - \dfrac{2}{3}u_n, \\ t_n = x_n - \dfrac{1}{2}u_n - \dfrac{1}{2}\dfrac{u_n}{1+\frac{3}{2}\left(\frac{f'(y_n)}{f'_n}-1\right)}, \\ x_{n+1} = t_n - \dfrac{f(t_n)}{H'_3(t_n)}, \end{cases} \tag{1}$$

where

$$u_n = \frac{f_n}{f'_n} \tag{2}$$

and

$$H'_3(t_n) = 2(f[x_n,t_n]-f[x_n,y_n]) + f[y_n,t_n] + \frac{y_n-t_n}{y_n-x_n}(f[x_n,y_n]-f'_n). \tag{3}$$

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http://dx.doi.org/10.1016/j.amc.2015.10.055
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**II. JHIF8**: This method is also based on Jarratt's optimal fourth order method [2] followed by a Newton-like step in which the function (instead of derivative) is replaced by Hermite interpolating polynomial. The method is given by

$$\begin{cases} y_n = x_n - \dfrac{2}{3}u_n, \\ t_n = x_n - \dfrac{1}{2}u_n - \dfrac{1}{2}\dfrac{u_n}{1+\frac{3}{2}\left(\frac{f'(y_n)}{f'_n}-1\right)}, \\ x_{n+1} = t_n - \dfrac{H_3(t_n)}{f'(t_n)}, \end{cases} \tag{4}$$

where

$$H_3(t_n) = f_n + f'_n\frac{(t_n-y_n)^2(t_n-x_n)}{(y_n-x_n)(x_n+2y_n-3t_n)} + f'(t_n)\frac{(t_n-y_n)(x_n-t_n)}{x_n+2y_n-3t_n} - f[x_n,y_n]\frac{(t_n-x_n)^3}{(y_n-x_n)(x_n+2y_n-3t_n)}. \tag{5}$$

**III. HK8**: This method is based on King's optimal fourth order method [3] followed by a modified Newton step replacing the function by Hermite interpolating polynomial. The method is given by

$$\begin{cases} y_n = x_n - u_n, \\ t_n = y_n - \dfrac{f(y_n)}{f'_n}\dfrac{f_n+\beta f(y_n)}{f_n+(\beta-2)f(y_n)}, \\ x_{n+1} = t_n - \dfrac{H_3(t_n)}{f'(t_n)}, \end{cases} \tag{6}$$

where $H_3(t_n)$ is given by (5).

**IV. WLN**: The method is based on Ostrowski's optimal fourth order method [4] and further developed by adding modified Newton step replacing the function by Hermite interpolating polynomial. The method is given by

$$\begin{cases} y_n = x_n - u_n, \\ t_n = y_n - \dfrac{f(y_n)}{f'_n}\dfrac{f_n}{f_n-2f(y_n)}, \\ x_{n+1} = t_n - \dfrac{H_3(t_n)}{f'(t_n)}, \end{cases} \tag{7}$$

where $H_3(t_n)$ is given by (5).

## 2. Order of convergence

Here, we prove that the order of convergence in all the above four methods is not optimal.

**Theorem 1.** *Let the function f(x) be sufficiently differentiable in a neighborhood of its zero $x^*$. If an initial approximation $x_0$ is sufficiently close to $x^*$, then the order of convergence of JHID8 and JHIF8 is seven and four, respectively.*

**Proof.** *Order of* JHID8:

Let $e_n = x_n - x^*$, $e_{y_n} = y_n - x^*$ and $e_{t_n} = t_n - x^*$ be the errors in the $n$th iteration. Using the fact that $f(x^*) = 0$ and $f'(x^*) \neq 0$, we write the Taylor's series expansion of the function $f(x_n)$ about $x^*$ to the required terms as follows

$$f(x_n) = f'(x^*)\left(\sum_{k=1}^{6} A_k e_n^k + O(e_n^7)\right), \tag{8}$$

where $A_k = (1/k!)f^{(k)}(x^*)/f'(x^*),\ k = 1, 2, 3, 4, \dots$

Also,

$$f'(x_n) = f'(x^*)\left(\sum_{k=1}^{6} kA_k e_n^{k-1} + O(e_n^6)\right). \tag{9}$$

Substitution of (8) and (9) in the first step of (1) yields

$$e_{y_n} = \frac{1}{3}e_n + \frac{2}{3}\sum_{k=2}^{6} L_{k-1}e_n^k + O(e_n^7), \tag{10}$$

where $L_1 = A_2,\ L_2 = -2(A_2^2 - A_3),\ L_3 = 4A_2^3 - 7A_2A_3 + 3A_4,\ L_4 = -2(4A_2^4 - 10A_2^2A_3 + 3A_3^2 + 5A_2A_4 - 2A_5), L_5 = 16A_2^5 - 52A_2^3A_3 + 33A_2A_3^2 + 28A_2^2A_4 - 17A_3A_4 - 13A_2A_5 - 5A_6$.

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