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Rogue waves in a resonant erbium-doped fiber system with higher-order effects

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ABSTRACT

We mainly investigate a coupled system of the generalized nonlinear Schrödinger equation and the Maxwell–Bloch equations which describes the wave propagation in an erbium-doped nonlinear fiber with higher-order effects including the forth-order dispersion and quintic non-Kerr nonlinearity. We derive the one-fold Darboux transformation of this system and construct the determinant representation of the n-fold Darboux transformation. Then the determinant representation of the nth new solutions $(E^{[n]}, p^{[n]}, \eta^{[n]})$ which were generated from the known seed solutions (E, p, η) is established through the n-fold Darboux transformation. The solutions $(E^{[n]}, p^{[n]}, \eta^{[n]})$ provide the bright and dark breather solutions of this system. Furthermore, we construct the determinant representation of the nth-order bright and dark rogue waves by Taylor expansions and also discuss the hybrid solutions which are the nonlinear superposition of the rogue wave and breather solutions.

1. Introduction

Recently, the long haul optical communication has attracted considerable interest of scientists around the world. But the efficiency of propagation of optical communication is still not very well. There are two important reasons, one is because of the dispersion and the other is due to the attenuation. In telecommunications, what we are interested is the variation of group velocity with frequency, because the absolute wave phase is often not important while the propagation of pulses is important. The dispersion makes the spread of the optical pulse temporally and may lead to the falling of the energy on the next bit slot. The dispersion is the linear effect for the propagation of optical pulse in fibers. The attenuation results from the optical losses which are the inherent feature of the optical fiber. The optical losses also cause the vanishing of the optical pulse due to the absorption and scattering [1].

What is important for the propagation of optical pulses in optical fibers is the nonlinear effect. The optical fiber behaves nonlinearity when the intensity of the optical pulse exceeds a certain threshold value. The most crucial effect is the self-phase modulation (SPM). While traveling in fibers, an optical pulse will induce a varying refractive index of the fiber due to the Kerr effect. This variation in refractive index will produce a phase shift in the pulse which leads to a change of the pulse's frequency spectrum. The spectral broadening process of SPM can balance with the temporal compression due to the anomalous dispersion and reach an equilibrium state when the pulse is of adequate intensity. The resulting pulse is called an optical soliton [2]. The possibility of the propagation of optical solitons which are governed by the nonlinear Schrödinger (NLS) equation was firstly introduced by Hasegawa and Tappert in 1973 [3,4]. Mollenauer et al. observed experimental solitons in low-loss fiber in 1980

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[5]. Another momentous nonlinear effect is the self-induced transparency (SIT). The SIT means that the coherent absorption and re-emission of pulses make the two-level medium optically transparent to this wavelength when the energy difference between the two levels of the medium matches with the optical wavelength. The optical pulse can be amplified and reshaped by passing through an active zone doped with resonant atoms like the erbium. The resonant interaction can neutralize the optical losses in fibers. McCall and Hahn put forward SIT solitons in a two-level resonant system in 1967, which is usually described by the Maxwell–Bloch (MB) equations [6].

Considering the optical fiber doped with resonant materials such as the erbium, which is governed by a coupled system of the NLS equation and the MB equations, the optical pulses satisfy both the NLS equation and the MB equations. It is necessary to amplify the optical pulses because of the decaying in the process of the propagation in fibers. In 1983 Maimistov and Manykin firstly proposed the NLS–MB system [7], after that Nakazawa and his cooperators observed SIT solitons in an erbium-doped silica fiber in 1991 [8] and more studies about the NLS–MB can be seen in Refs. [9–12].

Mitschke and Mollenauer found that the observed solitons in experiments did not match with the theoretical properties [13]. The propagation equation approximated to a second-order dispersion (group velocity dispersion) with a cubic Kerr nonlinearity (SPM). This leads to the difference between the experimentally observed soliton and NLS soliton in normal fibers. The difference resulted from the additional perturbation of higher order effects such as the higher-order dispersion, self steepening and higher-order nonlinear effects. Therefore the same difference will appear, if one experimentally studies the NLS–MB soliton in erbium-doped optical fibers. So it is necessary to investigate the propagation equation with higher-order effects. The generalized nonlinear Schrödinger (GNLS) equation is derived by Porsezian et al. [14]. The coupled generalized nonlinear Schrödinger and Maxwell–Bloch (GNLS–MB) system is described as [15,16]

$$E_z = i(E_{tt} + 2|E|^2E) + i\tau(E_{tttt} + 8|E|^2E_{tt} + 2E^2E^*_{tt} + 6E^*E_t^2 + 4|E_t|^2E + 6|E|^4E) + 2p, \tag{1.1a}$$

$$p_t = 2i\omega p + 2E\eta, \tag{1.1b}$$

$$\eta_t = -(Ep^* + pE^*), \tag{1.1c}$$

where subscripts z, t denote as partial derivatives with respect to the distance and time, the asterisk symbol as the complex conjugate, E as the normalized slowly varying amplitude of the complex field envelope, $p = v_1v_2^*$ as the polarization, and $\eta = |v_1|^2 - |v_2|^2$ as the population inversion with v_1 and v_2 representing the wave functions of the two energy levels of the resonant atoms, ω as the frequency. The more crucial reason to study the above system in this paper is because of the existence of the quintic non-Kerr nonlinear term which is more significant than the cubic Kerr nonlinearity because the non-Kerr nonlinearity is responsible for the stability of localized solutions [17,18].

In recent years, comparing with solitons, the study on rogue waves in optics has also attracted considerable research due to their potential applications in different branches of physics. The study started from the pioneering measurement of Solli et al. by analyzing super-continuum generations in optical fibers [19]. The rogue waves appear from nowhere and disappear without a trace, which is the description of the characteristics of rogue waves [20]. The rogue wave occurs for the modulation instability (MI) [21–25]. One of the possible generating mechanisms for rogue waves is the creation of breathers, then the larger rogue waves can be generated when two or more breathers collide [26]. The research on rogue waves has made many achievements among which Akhmediev has reported the recent progress in investigating optical rogue waves in Ref. [27].

To the best of our knowledge, there are few people to study the GNLS–MB system in Eq. (1.1) so far. The solitons and breather solutions of the GNLS–MB system have been partly established in Ref. [16], but the rogue waves of this system is still not reported by anybody. We will construct the determinant representation of the n-fold Darboux transformation of the GNLS–MB system, which is similar to the NLS–MB system [12] and H-MB system [28–30]. Then the nth-order rogue waves of the three optical fields will be given by determinants. Moreover, the p and η are found to be dark rogue waves.

The paper is organized as follows. In Section 2, the Lax pair of the GNLS–MB system is recalled, and we derive the one-fold Darboux transformation of the GNLS–MB system. In Section 3, the determinant representation of the n-fold Darboux transformation and formulas of $(E^{[n]}, p^{[n]}, \eta^{[n]})$ are expressed. In Section 4, the bright and dark breather solutions are generated from periodic seed solutions. In Section 5, we construct the determinant representation of the nth-order rogue wave by applying Taylor expansions and discuss the effects of parameter τ on the rogue wave solutions. Additionally, we discuss the hybrid solutions which are the nonlinear superposition of the rogue wave and breather solutions. Finally, we summarize the results in Section 6.

2. Lax pair and the one-fold Darboux transformation

In this section, we will derive the one-fold Darboux transformation of the GNLS–MB system in Eq. (1.1) of which the Lax pair is

$$\Psi_t = U\Psi, \tag{2.1a}$$

$$\Psi_z = V\Psi, \tag{2.1b}$$

where

$$\Psi = \begin{pmatrix} \Psi_1(\lambda;\, t,\, z) \\ \Psi_2(\lambda;\, t,\, z) \end{pmatrix}, \tag{2.2a}$$