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Matrix games with payoffs of belief structures

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ABSTRACT

Imprecise matrix games, such as interval-valued matrix games and fuzzy matrix games, have attracted much interest for a long time. Most of the previous studies on imprecise matrix games mainly focus on the fuzzy uncertainty of payoffs. However, the uncertainties of non-specificity and discord involved in payoffs are not well addressed so far. The purpose of this paper is to study the matrix game with such types of uncertainties. In order to achieve that purpose, we present a matrix game model with payoffs of belief structures so as to integrate discord and nonspecificity. The proposed model can be used to express more imprecise interactions between players in the reality. Besides, as another main contribution of the study, an effective method for solving matrix games with belief structures payoffs is developed to help us find the equilibrium points of the kind of games. At last, an example is given to illustrate the proposed model and method.

1. Introduction

Competitive situations arise in countless types of fields. Game theory provides a mathematical framework to explain and study the interactions of related individuals in interactive decision situations where the aims, goals and preferences of the participating agents are potentially in conflict [1–3]. A strategic game contains three parts: set of players, set of strategies for each player, and payoff for each combination of strategies from different players, respectively. Since its proposal, game theory has achieved a great success in multiple of disciplines and fields, typically including structure and dynamics of multilayer networks [4,5], epidemic spreading on complex networks [6], social dilemmas and opinion formation [7,8], information fusion and decision-making [9,10], understanding of cooperation and reciprocity on graphs [11–16], to name but a few.

The two-person zero-sum matrix game is one of the simplest games, often called a matrix game for short. In that game, there are only two players, and the gain of one player is definitely equal to the loss of the other. Payoffs of a matrix game are usually represented by a payoff matrix. Traditional, in matrix games payoffs are represented by crisp values, which indicate that they are precisely known. However, in reality, the determination of payoffs often depends on the assessment of observers to the environment and situation of the game. It is inevitably involved in subjective or objective uncertainty due to the lack of information, inability of human judgments, and drawbacks of assessment methods. Therefore, studying games with imprecise or uncertain payoffs becomes a very important research direction.

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Some studies have discussed the imprecise matrix games. One way to describe the imprecise payoffs is by interval data. In [17], Liu and Kao computed the value of the interval-valued matrix game by developing a pair of two-level mathematical programming models, in which the obtained value is also the interval. Li [18] has proved that an interval-valued matrix game can decompose into two matrix games with crisp value payoffs so that they can be solved by a classical linear programming approach based on the minimax theorem. Other approaches to solve interval-valued matrix games can also be found in [19–23]. Another compelling imprecise matrix game is the fuzzy matrix game. In that context, payoffs of games are represented by fuzzy numbers which are more vague than interval data. With respect to the fuzzy matrix game, hundreds of papers have studied how to solve it and obtain optimal strategies. For example, based on ranking functions of fuzzy numbers and auxiliary linear programming models, Campos [24] first proposed methods to solve fuzzy matrix games. Bector et al. [25] and Vijay et al. [26] transformed the fuzzy matrix games into two crisp linear programming problems. In [27], Maeda characterized the equilibrium strategy of fuzzy matrix games through the fuzzy max order. Li [28] proposed a two-level linear programming method for solving matrix games with payoffs of triangular fuzzy numbers. While in Li's model, the value of the fuzzy matrix game cannot be assured to always be a triangular fuzzy numbers, in [29], the original model has been improved. Recent research about fuzzy matrix games can be found in [30,31]. Apart from interval values and fuzzy numbers, matrix games with payoffs of fuzzy random variables [32], and interval-valued intuitionistic fuzzy sets [33] have also been given much attention.

Regarding uncertainty, Klir and Yuan [34] have identified three basic types: fuzziness, discord, and nonspecificity. Several branches of theories have been proposed to deal with these kind of uncertainties, such as fuzzy set theory, rough set theory, possibility theory, and so forth. Fuzziness stands for the unclearness or indistinctness about objects' characters. Since been proposed by Zadeh in 1960s, fuzzy set theory [35] has almost become a paradigmatic framework for representing fuzzy, ambiguous, vague, and linguistic information, and attracted lots of interest from many fields, such as multiple criteria decision-making [36,37], job-shop scheduling [38], environmental impact assessment [39], fuzzy traveling salesman problem [40,41], etc. [42–44]. Discord mainly stands for conflicting ideas to the same object, and nonspecificity means the diversity of possible results. The discord and nonspecificity are also unified as ambiguity in Jousselme et al.'s research [45]. To deal with the ambiguity, Dempster–Shafer evidence theory [46,47] are usually used. As a useful tool of uncertainty reasoning, this theory provides a representation of uncertain information called as belief structures which can contain discord and nonspecificity at the same time. As an open issue, the measure of total uncertainty in belief structures has been given much attention [34,45,48,49].

As mentioned above, whether payoffs are represented by interval data or fuzzy numbers, most research has mainly focused on the fuzziness of payoffs, while nonspecificity and discord are not given much interest. Therefore, this study focuses on imprecise matrix games with nonspecificity and discord types of uncertainties. In order to effectively represent the nonspecificity and discord, we utilize belief structures from Dempster–Shafer evidence theory to model imprecise payoffs. A method is proposed to solve the matrix game with belief structure payoffs. And to illustrate the proposed method, a fictitious example is provided at last.

The rest of this paper is organized as follows. Section 2 gives a brief introduction about Dempster–Shafer theory and matrix games. Section 3 presents the proposed method for solving matrix games with belief structure payoffs. In Section 4, a fictitious example is given to illustrate the proposed method. Section 5 concludes this paper.

2. Preliminaries

2.1. Dempster–Shafer evidence theory

Dempster–Shafer evidence theory, also called Dempster–Shafer theory (D–S theory) or evidence theory, is famous for its ability to represent and handle uncertain information. D–S theory is often regarded as an extension of the Bayesian theory because it has an advantage of directly expressing the "uncertainty" by assigning the probability to the set composed of multiple objects. This theory was proposed by Dempster [46] and developed by Shafer [47] in 1960s–1970s. Later, Smets [50] proposed transferable belief model (TBM) to improve the D–S theory. In recent years, many new ideas have been proposed to further improve and enrich this theory. For example, Yang et al. [51] have given a belief rule base to extend classical fuzzy rule base based on D–S theory, and presented an evidential reasoning rule [52] for improving the results of evidence combination. Deng [53] proposed a generalized evidence theory (GET) to study the conflict management in the open world. Based on D–S theory, a new model called D numbers [54–57] has been proposed to represent uncertainty, which has attracted much interest [58–60]. Besides, as an extension of this theory, Dezert–Smarandache theory of plausible and paradoxical reasoning [61,62] has been given much attention. In the study of D–S theory, several key issues, such as evidence combination [63–65], construction of belief structures [66], are also widely concerned.

In D–S theory [46,47], a problem domain denoted by a finite nonempty set Ω of mutually exclusive and exhaustive hypotheses is called the frame of discernment. Let 2^Ω denote the power set of Ω, the elements of 2^Ω are called propositions. Given a frame of discernment Ω, a belief structure is a mapping $m : 2^\Omega \to [0, 1]$, such that

$$m(\emptyset) = 0 \quad \textit{and} \quad \sum_{A \in 2^\Omega} m(A) = 1 \tag{1}$$

A belief structure is also called a mass function or a basic probability assignment (BPA). Associated with each belief structure is the belief measure and plausibility measure, *Bel* function and *Pl* function, respectively. The belief function $Bel : 2^\Omega \to [0, 1]$ is