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Pareto optimization scheduling with two competing agents to minimize the number of tardy jobs and the maximum cost

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ABSTRACT

This paper investigates the Pareto optimization scheduling problem on a single machine with two competing agents A and B in which agent A wants to minimize the number of tardy A-jobs and agent B wants to minimize the maximum cost of B-jobs. In the literature, the constrained optimization problem of minimizing the number of tardy A-jobs under the restriction that the maximum cost of B-jobs is bounded is solved in polynomial time. This implies that the corresponding Pareto optimization scheduling problem can be solved in a weakly polynomial time. In this paper, by presenting a new algorithm for the constrained optimization problem, we provide a strongly polynomial-time algorithm for the corresponding Pareto optimization scheduling problem. Experimentation results show that the proposed algorithm for the considered problem is efficient.

1. Introduction

In the single-machine two-agent scheduling, two agents A and B, each with a set of nonpreemptive jobs, compete to process its own jobs on a common machine in order to minimize its own objective function.

Let $\mathcal{J}^A = \{J_1^A, J_2^A, \ldots, J_{n_A}^A\}$ and $\mathcal{J}^B = \{J_1^B, J_2^B, \ldots, J_{n_B}^B\}$ denote the job sets of agent A and agent B, respectively. Let $\mathcal{J} = \mathcal{J}^A \cup \mathcal{J}^B$ and $n = n_A + n_B$. For each $X \in \{A, B\}$, the jobs in $\mathcal{J}^X$ are called X-jobs and each subset of $\mathcal{J}^X$ is called an X-set. The processing time and due date of job J_j^X are denoted by p_j^X and d_j^X, respectively, $j = 1, 2, \ldots, n_X$. All jobs are available at time 0 and a feasible schedule processes the jobs without overlap. In a feasible schedule, the jobs with processing time 0 can always be processed at time 0. Then we assume that $p_j^X > 0$ for all X and j. Given a feasible schedule σ, we denote by $S_j^X(\sigma)$ and $C_j^X(\sigma)$ the starting time and the completion time of job J_j^X, respectively. If $C_j^X(\sigma) \le d_j^X$, then we call J_j^X an *early job* and define that $U_j^X(\sigma) = 0$. Otherwise, we call J_j^X a *tardy job* and define that $U_j^X(\sigma) = 1$. We assume in this paper that all processing times, due dates, and objective values are integral.

Let f^A and f^B be the objective functions of agent A and agent B, respectively, to be minimized. In the two-agent scheduling research, the constrained optimization and the Pareto optimization are widely considered.

Constrained optimization problem (CP): Following the notation introduced by Agnetis et al. [1], the two constrained optimization scheduling problems can be denoted by $1||f^A : f^B \le Q$ and $1||f^B : f^A \le Q$. In problem $1||f^A : f^B \le Q$, we want to find a

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schedule σ so that $f^A(\sigma)$ is minimized under the restriction that $f^B(\sigma) \le Q$. Alternatively, in problem $1||f^B : f^A \le Q$, we want to find a schedule σ so that $f^B(\sigma)$ is minimized under the restriction that $f^A(\sigma) \le Q$.

Pareto optimization problem (PP): For a given schedule π, we denote by $(f^A(\pi), f^B(\pi))$ the objective vector of π. If there exists no schedule σ such that $(f^A(\sigma), f^B(\sigma)) \le (f^A(\pi), f^B(\pi))$ and at least one of the two strict inequalities $f^A(\sigma) < f^A(\pi)$ and $f^B(\sigma) < f^B(\pi)$ holds, we call π a *Pareto optimal schedule* and $(f^A(\pi), f^B(\pi))$ the *Pareto optimal point* corresponding to π. The goal of Pareto optimization scheduling is to find all Pareto optimal points and, for each Pareto optimal point, a corresponding Pareto optimal schedule. Following the notation introduced by Agnetis et al. [1], the two-agent Pareto optimization scheduling problem on a single machine to minimize f^A and f^B can be denoted by $1||f^A \circ f^B$.

In this paper, we study the (PP) problem $1||f^A \circ f^B$ with $f^A(\sigma) = TU^A(\sigma)$ and $f^B(\sigma) = f^B_{\max}(\sigma)$, where $TU^A(\sigma) = \sum_{j=1}^{n_A} U^A_j(\sigma)$ and $f^B_{\max}(\sigma) = \max\{f^B_1(C^B_1(\sigma)), f^B_2(C^B_2(\sigma)), \dots, f^B_{n_B}(C^B_{n_B}(\sigma))\}$ with $f^B_j(\cdot)$ being a nondecreasing function for each $j = 1, 2, \dots, n_B$. In a schedule, all tardy A-jobs can be scheduled after all early A-jobs and all B-jobs. This means that only early A-jobs and all B-jobs can be cared in the schedule. For simplicity, the tardy A-jobs will not be considered in the schedules.

The model of two-agent scheduling was first introduced by Baker and Smith [3] and Agnetis et al. [1]. The objective functions considered in their research include the maximum of regular functions (e.g., makespan or maximum lateness), the total (weighted) completion time, and the number of tardy jobs. Some incorrect results in Baker and Smith [3] were pointed out and corrected by Yuan et al. [21]. Ng et al. [17] considered the problem that minimizes the total completion time for one agent given that the number of tardy jobs of the other agent is bounded. Agnetis et al. [2] initially developed a combination approach for two-agent scheduling problems. Cheng et al. [4,5] extended the two-agent setting to the multi-agent setting in which there are more than two agents. Leung et al. [14] and Elvikis and T'kindt [6] considered several two-agent scheduling problems on parallel machines. Kovalyov et al. [12], Li and Yuan [15], and Fan et al. [7] introduced the two-agent scheduling problems to the batching model and designed different dynamic programming algorithms. More extensive discussion on multi-agent scheduling can be found in Perez-Gonzalez and Framinan [18].

Agnetis et al. [1] presented a comprehensive research for the two-agent constrained optimization scheduling problems and the two-agent Pareto optimization scheduling problems. Especially, Agnetis et al. [1] provided an $O(n_A \log n_A + n_B \log n_B) = O(n \log n)$-time algorithm for the (CP) problem $1||TU^A : f^B_{\max} \le Q$ under the assumption that the inverse function of $f^B_j(\cdot)$ is available for each $j \in \{1, 2, \dots, n_B\}$. Although the (PP) problem $1||TU^A \circ f^B_{\max}$ was not addressed in Agnetis et al. [1], their $O(n\log n)$-time algorithm for $1||TU^A : f^B_{\max} \le Q$ in fact implies that the (PP) problem $1||TU^A \circ f^B_{\max}$ can be solved in a weaker polynomial $O(n_A n \log n \log Q^*)$-time, where $Q^* = \max\{f^B_j(P) : 1 \le j \le n_B\}$ with $P = p^A_1 + \dots + p^A_{n_A} + p^B_1 + \dots + p^B_{n_B}$ is an obvious upper bound of $f^B_{\max}$. The principle for this observation can be understood by the arguments below the definition of (PP) problem in Agnetis et al. [1]: There are at most $n_A + 1$ Pareto optimal points, and each Pareto optimal point can be obtained by binary-search by running the above $O(n\log n)$-time algorithm $O(\log Q^*)$ times.

In this paper we revisit the (PP) problem $1||TU^A \circ f^B_{\max}$ and provide an algorithm to solve the problem in $O(\min\{n_A, n_B\} n_A^2 (n \log n + n_B^2)) = O(n^5)$ time which is strongly polynomial. Throughout this paper, the A-jobs $\mathcal{J}^A$ are sorted in the EDD order $(J^A_1, J^A_2, \dots, J^A_{n_A})$ so that $d^A_1 \le d^A_2 \le \dots \le d^A_{n_A}$, which costs $O(n_A \log n_A)$ time. Then we fix this order $(J^A_1, J^A_2, \dots, J^A_{n_A})$ in our discussion and call it the *FEDD order* of A-jobs. An FEDD-schedule of an A-set $\tilde{\mathcal{J}}^A$ is the schedule in which the jobs of $\tilde{\mathcal{J}}^A$ are scheduled in the order coincide with the FEDD order of the A-jobs. Moreover, as in Agnetis et al. [1], we assume that the inverse function of $f^B_j(\cdot)$ is available for each $j \in \{1, 2, \dots, n_B\}$.

The rest of the paper is constructed as follows. In Section 2, we present the principle of Pareto optimization scheduling used in this paper, describe two algorithms related to the (CP) problems $1||TU^A : f^B_{\max} \le Q$ and $1||f^B_{\max} : TU^A \le 0$ to be used as subroutines, and provide an algorithm, called Pareto(F), which generates a schedule optimal for $1||TU^A : f^B_{\max} \le F$ and Pareto optimal for $1||TU^A \circ f^B_{\max}$. Then the remaining matter is to determine the time complexity of algorithm Pareto(F) which is done in Section 3. To this end, we discuss in Section 3.1 the scheduling of A-jobs subject to blocking intervals and in Section 3.2 the effects to schedule the B-jobs as blocking intervals. In Section 3.3, an algorithm, called MIN(A, Q), for solving problem $1||TU^A : f^B_{\max} \le Q$ is presented. Finally, in Section 3.4, we show that algorithm Pareto(F) can runs in $O(n^4)$ time. As a consequence, the (PP) problem $1||TU^A \circ f^B_{\max}$ can be solved in $O(n^5)$ time. We give some numerical experiments to support the conclusions in Section 4.

2. Principles and algorithms

We present in Section 2.1 the principle of Pareto optimization scheduling used in this paper, and in Section 2.2 two algorithms related to the (CP) problems $1||TU^A : f^B_{\max} \le Q$ and $1||f^B_{\max} : TU^A \le 0$. In Section 2.3, we provide an algorithm, called Pareto(F), which generates a schedule optimal for $1||TU^A : f^B_{\max} \le F$ and Pareto optimal for $1||TU^A \circ f^B_{\max}$.

2.1. Pareto optimization

Consider the Pareto optimization scheduling problem $1|\bullet|f \circ g$ on a single machine to minimize two objective functions f and g. We assume that $f(\sigma)$ and $g(\sigma)$ are integral for every schedule σ. A schedule π is called an *optimal and Pareto optimal schedule* of the (CP) problem $1|\bullet|f : g \le V$ if π is optimal for problem $1|\bullet|f : g \le V$ and also Pareto optimal for problem $1|\bullet|f \circ g$. Then the following lemma can be observed.