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Finite dimensional realization of a quadratic convergence yielding iterative regularization method for ill-posed equations with monotone operators

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ABSTRACT

Recently Jidesh et al. (2015), considered a quadratic convergence yielding iterative method for obtaining approximate solution to nonlinear ill-posed operator equation $F(x) = y$, where F: $D(F) \subseteq X \to X$ is a monotone operator and X is a real Hilbert space. In this paper we consider the finite dimensional realization of the method considered in Jidesh et al. (2015). Numerical example justifies our theoretical results.

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1. Introduction

Let X be a real Hilbert space and let F: $D(F) \subseteq X \to X$ be a nonlinear monotone operator, i.e.,

$$\langle F(u) - F(v), u - v\rangle \text{ for all } u, v \in D(F). \tag{1.1}$$

Throughout this paper $\langle ., .\rangle$ and $\|.\|$, respectively, denote the inner product and the corresponding norm in X.

This paper is concerned with the follow up of the paper [5] where we considered an iterative regularization method for approximately solving the ill-posed operator equation

$$F(x) = y. \tag{1.2}$$

Many inverse problems of practical interest lead to mathematical problems which can be modeled to the Eq. (1.2). It is assumed that (1.2) has a solution, namely $\hat{x}$ and we assume that only noisy data y^δ are available, such that

$$\|y - y^\delta\| \le \delta.$$

Then the problem of recovery of $\hat{x}$ from noisy equation $F(x) = y^\delta$ is ill-posed, in the sense that small perturbation in the data can cause large deviation in the solution [8,9–11]. For solving (1.2) with monotone operator one usually uses the Lavrentiev regularization method (see [9],[10]). In this method the regularized approximation x_α^δ is obtained by solving the operator equation

$$F(x) + \alpha(x - x_0) = y^\delta. \tag{1.3}$$

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Here, x_0 is the known initial guess which plays the role of a selection criterion ([10]). It is known (cf. [10], Theorem 1.1) that the Eq. (1.3) has a unique solution x_α^δ for $\alpha > 0$, provided F is Fréchet differentiable and monotone in the ball $B_r(\hat{x}) \subset D(F)$ with radius $r = \|\hat{x} - x_0\| + \frac{\delta}{\alpha}$. Here and below $B_r(x)$ and $\overline{B_r(x)}$, respectively stand for open and closed balls in X with center $x \in X$ and radius $r > 0$. If F is globally monotone, i.e., $D(F) = X$ in (1.1), then by Browder–Minty theorem [3], (1.3) has a unique solution x_α^δ for all x_0, $y^\delta \in X$. However the regularized Eq. (1.3) remains nonlinear and one may have difficulties in solving them numerically.

In [4], George and Elmahdy considered the method

$$x_{n+1,\alpha}^{h,\delta} = x_{n,\alpha}^{h,\delta} - R_\alpha^{-1}(x_{n,\alpha}^{h,\delta})P_h[F(x_{n,\alpha}^{h,\delta}) - y^\delta + \alpha(x_{n,\alpha}^{h,\delta} - x_0)] \tag{1.4}$$

where $R_\alpha(x) := P_hF'(x)P_h + \alpha I$, $\{P_h\}_{h>0}$ be a family of orthogonal projections of X onto $R(P_h)$, the range of P_h and $x_{0,\alpha}^{h,\delta} := P_hx_0$. Convergence analysis using majorizing sequence in [4] is bit more cumbersome with less details. In this paper we give a simple proof for the convergence of $x_{n,\alpha}^{h,\delta}$ to $\hat{x}$ with more details.

In this study first we prove that

$$P_hFP_h(x) + \alpha P_h(x - x_0) = P_hy^\delta \tag{1.5}$$

has a unique solution $x_\alpha^{h,\delta}$ in $B_r(P_hx_0)$ and then we prove that the sequence $(x_{n,\alpha}^{h,\delta})$ defined in (1.4) converges quadratically to $x_\alpha^{h,\delta}$.

The rest of the paper is organized as follows. Convergence analysis of the method (1.4) is given in Section 2, error analysis and parameter choice strategy are given in Section 3. The implementation of the method (1.4) is given in Section 4. Finally a numerical example is given in the concluding Section 5.

2. The Method and its convergence

Let $\{P_h\}_{h>0}$ be a family of orthogonal projections of X onto $R(P_h)$, the range of P_h. Our aim in this section is to obtain an approximation for x_α^δ, in the finite dimensional space $R(P_h)$. For the results that follow, we impose the following conditions. Let

$$\epsilon_h := \|F'(x)(I - P_h)\|, \quad \forall x \in D(F)$$

and

$$b_h := \|(I - P_h)\hat{x}\|.$$

We assume that $\lim_{h\longrightarrow 0}\epsilon_h = 0$ and $\lim_{h\to 0} b_h = 0$ as $h \to 0$. The above assumption is satisfied if $P_h \to I$ point-wise and if $F'(x)$ is a compact operator. Further we assume that there exist $\varepsilon_0 > 0, b_0 > 0$ and $\delta_0 > 0$ such that $\epsilon_h < \epsilon_0$ and $b_h < b_0$.

2.1. Projection method

We consider the sequence $(x_{n,\alpha}^{h,\delta})$ defined in (1.4) for obtaining an approximation for x_α^δ in the finite dimensional subspace $R(P_h)$ of X. Note that iteration (1.4) is the finite dimensional realization of the method considered in [5].

We use the following assumptions for the convergence analysis.

Assumption 2.1. There exists a constant $k_0 \geq 0$ such that for every $x, u \in D(F)$ and $v \in X$ there exists an element $\Phi(x, u, v) \in X$ such that $[F'(x) - F'(u)]v = F'(u)\Phi(x, u, v)$, $\|\Phi(x, u, v)\| \leq k_0\|v\|\|x - u\|$.

Assumption 2.2. There exists a continuous, strictly monotonically increasing function φ: $(0, a] \to (0, \infty)$ with $a \geq \|F'(x_0)\|$ satisfying;

(i) $\lim_{\lambda\to 0}\varphi(\lambda) = 0$,
(ii) $\sup_{\lambda\geq 0}\frac{\alpha\varphi(\lambda)}{\lambda+\alpha} \leq c\varphi(\alpha)\ \forall\lambda \in (0, a]$ and
(iii) there exists $v \in X$ with $\|v\| \leq 1$ such that

$$x_0 - \hat{x} = \varphi(F'(x_0))v. \tag{2.1}$$

Let

$$r \geq 2(r_0 + \max\{1, \|\hat{x}\|\}) \text{ with } r_0 := \|\hat{x} - x_0\|.$$

Hereafter, we assume that $\epsilon_h \in (0, \epsilon_0)$, $\delta \in [0, \delta_0]$, $a_0 \geq \varepsilon_0 + \delta_0$, $\alpha \in (\delta + \epsilon_h, a_0)$.

Proposition 2.3. *Let F be a monotone operator and $\{P_h\}_{h>0}$ be a family of orthogonal projections of X onto $R(P_h)$ with $R(P_h) \subset D(F)$. Then P_hFP_h is a monotone operator on X and the operator Eq. (1.5) has a unique solution $x_\alpha^{h,\delta}$ for all x_0, $y^\delta \in X$. Furthermore $x_\alpha^{h,\delta} \in B_r(P_hx_0)$.*

Proof. Since F is monotone and $R(P_h) \subset D(F)$, we have

$$\langle P_hFP_h(x) - P_hFP_h(y), x - y\rangle = \langle F(P_hx) - F(P_hy), P_hx - P_hy\rangle \geq 0.$$

That is P_hFP_h is monotone operator and $D(P_hFP_h) = X$. Therefore by Browder–Minty theorem [3, Theorem 1.15.21], $R(P_hFP_h + \alpha I) = X$ (see also [6]) and $P_hFP_h + \alpha I$ is injective. Hence the equation

$$(P_hFP_h + \alpha I)x = P_h(y^\delta + \alpha x_0)$$

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