Applied Mathematics and Computation 273 (2016) 1234–1245

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Consensus seeking over Markovian switching networks with time-varying delays and uncertain topologies

Yilun Shang*

Department of Mathematics, Tongji University, Shanghai 200092, China

ARTICLE INFO

MSC:
93E03
93D05

Keywords:
Consensus
Multi-agent system
Markov process
Time delay
Uncertainty

ABSTRACT

Stochastic consensus problems for linear time-invariant multi-agent systems over Markovian switching networks with time-varying delays and topology uncertainties are dealt with. By using the linear matrix inequality method and the stability theory of Markovian jump linear system, we show that consensus can be achieved for appropriate time delays and topology uncertainties which are not caused by the Markov process, provided the union of topologies associated with the positive recurrent states of the Markov process admits a spanning tree and the agent dynamics is stabilizable. Feasible linear matrix inequalities are established to determine the maximal allowable upper bound of time-varying delays. Numerical examples are given to show the feasibility of theoretical results.

1. Introduction

Cooperative control of networked multi-agent systems has received increasing attention during the past few years, mainly due to wide applications of multi-agent systems in many areas such as flocking/swarming, formation control, attitude alignment, parallel computing, and distributed sensor fusion. In multi-agent cooperative control, an important topic is consensus (synchronization or agreement) which refers to steering a specific variable of group members to a common value across the network by using the local information, which is determined by the underlying network topology; see the surveys in [1,2]. Thus, the design of appropriate control protocols and algorithms for a group of dynamic agents seeking to reach consensus on certain quantities of interest is a critical problem.

The theoretic foundation of consensus control design for agents with simple single/double-integrator dynamics on deterministic (fixed or switching) topologies has been well understood so far through the application of graph-theoretical tools (see e.g. [3–5]). In many practical systems, the communication link between the agents may only be available at random times due to link/node failure, signal losses, or packet drops; this motivates the recent investigation of consensus over random graphs [6–15]. For example, Hatano and Mesbahi [7] considered the asymptotic agreement of a continuous-time single-integrator agent dynamics over classical undirected random graphs, where each information channel between a pair of agents exists independently at random. The results were further extended by Porfiri and Stilwell [8] to solve mean square consensus problem on directed and weighted random information networks. Shang [9] addressed multi-agent coordination in directed moving neighborhood random networks generated by random walkers. Tahbaz-Salehi and Jadbabie [10] provided necessary and sufficient conditions for almost sure consensus of a group of single-integrator agents, where the communication graph was derived from a strictly stationary ergodic graph process. Matei et al. [11] discussed the consensus problem of both discrete-time and

* Fax: +86-21-65981384.
E-mail address: shylmath@hotmail.com, shyl@tongji.edu.cn

http://dx.doi.org/10.1016/j.amc.2015.08.115
0096-3003/

continuous-time multi-agent systems with single-integrator dynamics over Markovian switching topologies; they showed that the system achieves average consensus almost surely if and only if the union of topologies corresponding to the states of the Markov process is strongly connected. Similar results were obtained for double-integrator agent dynamics by Miaoet al. [12], and more realistic aspects including measurement noises as well as quantization errors were factored in by Huang et al. [13]. Group consensus of discrete-time and continuous-time multi-agent systems with Markovian switching topologies was recently discussed by Zhao and Park [14] and Shang [15], respectively. Besides, means square consensus problems over fixed topology with communication noises were tackled for discrete-time and continuous-time linear time-invariant systems in [16,17], respectively.

It is noted that for the agent dynamics described by more general linear time-invariant systems, it is challenging to derive consensus conditions for achieving consensus due to the possible existence of strictly unstable poles in the open-loop matrix; see e.g. [18,19]. Recently, Youet al. [20] extended the results of [11] to linear time-invariant systems; they established almost sure convergence by utilizing the linear dynamics governing the evolution of the mean square consensus error. In [21], network-induced delay and random noise effect were tackled in-depth drawing on the stability analysis of differential delay equations. Graphic conditions for group consensus were discussed by Shang [22] for linear time-invariant systems under Markovian switching topologies. The stochastic consensus of linear multi-input and multi-output systems with communication noises and Markovian switching topologies was studied in [23]. However, the systems studied in [20,22,23] are without time delay. It is well recognized that unmodelled time delay may affect the performance and causes instability of a system [24,25]. In multi-agent systems, time-varying delays arise naturally due to the asymmetry of interactions, the congestion of the communication channels, and the finite transmission speed. Moreover, the system uncertainties exist in many situations; see e.g. [26–28]. Thus, this paper is devoted to deriving consensus conditions for linear multi-agent systems on Markovian switching topologies in the presence of both time-varying delays and topology uncertainties which are not related to the Markov process. Although delay robustness has been addressed in [12] for Markovian jump linear systems, their methods are mainly restricted to discrete-time systems with fixed communication delays.

This paper deals with the consensus problem of a group of agents with continuous-time linear dynamics, whose communication topology is a randomly switching network driven by a time-homogenous Markov process. Each communication pattern (i.e., directed graph) corresponds to a state of the Markov process. Due to the introduction of time-varying delays and uncertainties, the present approaches in [11–13,20,22,23] do not apply. Here, we show how the linear matrix inequality (LMI) method, together with results inspired by stability analysis of Markovian jump linear systems, can be used to prove stochastic consensus results. It is shown that the multi-agent system can reach mean square and almost sure consensus for appropriate time-varying delays and topology uncertainties if the union of the topologies corresponding to the positive recurrent states of the Markov process has a spanning tree and the agent dynamics is stabilizable. Our results are presented in terms of feasible linear matrix inequalities, from which the maximal allowable upper bound of time-varying delays and uncertainties can be easily obtained by using Matlab's LMI Toolbox. The consensus gain is designed via a Riccati inequality, and the speed of consensus is also estimated. Finally, we work out some numerical examples to illustrate the availability of our theoretical results. We mention that the LMI method was used in [29] for system with random delays governed by a Markov chain, where the communication topology is, nevertheless, fixed.

We mention that another topic closely related to consensus is the synchronization of complex networks, where the synchronization stability of a network of oscillators is usually studied by using the master stability function method. The difference between synchronization and consensus is that the former analyzes the case that the uncoupled systems have identical nonlinear node dynamics which constitutes the ultimate synchronous trajectory, while the latter focuses on reaching an agreement on some variable of interest through local interactions. Synchronization can be viewed as a generalization of consensus to encompass nonlinear dynamics. Recent works along this line include [30] and [31], where finite-time synchronization for complex networks with Markov jump topology is studied. The LMI method was also used in [30] to determine sufficient synchronization conditions. But the system therein is without time delay.

The rest of the paper is organized as follows. Section 2 contains the problem formulation. Section 3 presents the main results. Section 4 gives simulation results and Section 5 concludes the paper.

Throughout this paper, the wildcard $*$ represents the elements below the main diagonal of a symmetric matrix. $\mathbf{1}_n$ and $\mathbf{0}_n$ mean the n-dimensional column vectors of all ones and all zeros, respectively. I_n is an $n \times n$ identity matrix. We often suppress the subscript n when the dimension is clear from the context. We say $A > B$ ($A \geq B$) if $A - B$ is positive definite (semi-definite), where A and B are symmetric matrices of same dimensions. A^T means the transpose of the matrix A. For a vector x, $\|x\|$ refers to its Euclidean norm. The set of real numbers is denoted by $\mathbb{R}$. Let 1_E signify the indicator function of an event E. By $A \otimes B$ we denote the Kronecker product of two matrices A and B, which admits the following useful properties: $(A \otimes B)(C \otimes D) = AC \otimes BD$, $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$, and $(A \otimes B)^T = A^T \otimes B^T$.

2. Problem formulation

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ represent a weighted directed graph of order N, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the set of nodes, i.e., agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges. A directed edge from node v_i to node v_j is denoted as an ordered pair (v_i, v_j), indicating that the information can be sent from agent v_i to agent v_j. The weighted adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ is defined by $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. $d_i^{in} = \sum_{j=1}^N a_{ij}$ and $d_i^{out} = \sum_{j=1}^N a_{ji}$ are called in-degree and out-degree of agent v_i, respectively. $\mathcal{G}$ is said to be balanced if $d_i^{in} = d_i^{out}$ for all $i = 1, \ldots, N$ [3]. The graph Laplacian matrix associated with the graph $\mathcal{G}$