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Modulus-based matrix splitting iteration methods for a class of nonlinear complementarity problem

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ABSTRACT

Some modulus-based matrix splitting iteration methods for a class of nonlinear complementarity problem are presented, and convergence analyses of the methods are given. Numerical experiments confirm the theoretical analysis, and show that the proposed methods are efficient.

1. Introduction

Complementarity problems have extensive applications in mechanical engineering, fluid flow through porous media, contact problem in elasticity, economic transportation and other areas of study.

We consider the following nonlinear complementarity problem:

$$u \geq 0, v = Mu + q + A(u) \geq 0, (u, v) = 0, \tag{1}$$

where $M \in R^{n \times n}$, $q \in R^n$, and the nonlinear term $A(u) = (A_1(u_1), \ldots, A_n(u_n))^T$, satisfying that $\frac{\partial A_i(u_i)}{\partial u_i} \geq 0$. About some information of this model, please see [14] and the references therein.

Using the change of variables, Noor(1988) characterized (1) as a fixed point problem, and developed some algorithms [16].

Some multisplitting methods for linear system were proposed by Frommer [12]. An advantage of these methods is that one only needs to compute some components according to the block property, another advantage is that it is suitable to parallel computation. Then Bai [1–4] presented a unified framework for the construction of various matrix multisplitting iterative methods for large sparse system of linear or nonlinear equations and analysed their convergence. Moreover, Bai [5–8] extended to solve the linear complementarity problem, and constructed some synchronous and asynchronous matrix splitting methods.

Recently, modulus-based iteration methods for solving the LCP problem by Bai [9], are very effective in practical application. Then, various kinds of modulus-based multisplitting iteration methods are developed, see [10,18–20] and the references therein.

In this paper, firstly we reformulate (1) as an implicit fixed-point equation. Then we construct some modulus-based matrix splitting iteration methods for solving (1). Under suitable conditions, we will prove the convergence of the modulus-based splitting iteration methods.

The outline of the paper is as follows. In Section 2, we present some necessary notations and useful lemmas. In Section 3, we establish the modulus-based matrix splitting iteration methods, In Section 4, the convergence of the new methods is proved. The numerical results are shown and discussed in Section 5.

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2. Preliminaries

We shortly introduce the necessary definitions and some lemmas, see [11–13,15,17].

Given two matrices $M = (m_{ij}),\ B = (b_{ij}) \in R^{n\times n}$. Then $M \geq B(M > B)$, iff $m_{ij} \geq b_{ij}(m_{ij} > b_{ij})$ holds for all $1 \leq i \leq n$, $1 \leq j \leq n$. If O is a null matrix and $M \geq O(M > O)$, M is called a nonnegative matrix. $|M|$ denotes the nonnegative matrix with entries $|m_{ij}|$.

Let M be a real $n \times n$ matrix. Its comparison matrix $\langle M\rangle = (\langle m_{ij}\rangle) \in R^{n\times n}$, where

$$\langle m\rangle_{ij} = \begin{cases} |m_{ij}|, i = j \\ -|m_{ij}|, i \neq j \end{cases}, \quad i, j = 1, 2, \ldots, n. \tag{2}$$

The matrix M is called an M-matrix if its off-diagonal entries are all non-positive and $M^{-1} \geq O$; M is called an H-matrix if its comparison matrix $\langle M\rangle$ is an M-matrix; M is called an H_+-matrix if it is an H-matrix with positive diagonal entries; if M is an M-matrix, Ω is a positive diagonal matrix, then $M \leq B \leq \Omega$ implies that B is an M-matrix.

Given $M = F - G$, if F is a non-singular matrix, then $M = F - G$ is called a splitting of the matrix M; for the splitting $M = F - G$, if $\langle M\rangle = \langle F\rangle - |G|$, the splitting is called an $H-$compatible splitting. If $M = F - G$ is an $H-$compatible splitting, then $\rho(F^{-1}\cdot G) \leq \rho(\langle F\rangle^{-1}\cdot G) \leq 1$.

Lemma 2.1 [11]. *Let $M \in R^{n\times n}$ be an $H-$matrix, $D = diag(M)$, $B = D - M$, then*

1. *Matrix M is non-singular;*
2. $|M^{-1}| \leq \langle M\rangle^{-1}$;
3. *$|D|$ is nonsingular and $\rho(|D|^{-1}\cdot|B|) < 1$.*

Lemma 2.2 [11]. *Let $M \in R^{n\times n}$ be a irreducible nonnegative matrix, then*

1. *The spectral radius $\rho(M)$ is a positive real eigenvalue of matrix M;*
2. *The eigenvector x of $\rho(M)$ is a positive vector;*
3. *$\rho(M)$ is a simple eigenvalue of M;*
4. *If any element of matrix M is increased, $\rho(M)$ increases.*

The following theorem provides an equivalent expression of the nonlinear complementarity Problem (1), which is attributed to establishing new matrix splitting iteration methods for solving the Problem (1).

Theorem 2.1. *Let $M = F - G$ be a splitting of the matrix $M \in R^{n\times n}$, and Ω be a positive diagonal matrix. For the nonlinear complementarity Problem (1), the following statements hold true:*

(a) *If (u, v) is a solution of the nonlinear complementarity Problem (1), then $x = \frac{h}{2}(u - \Omega^{-1}v)$ satisfies the implicit fixed-point equation*

$$(\Omega + F)x = Gx + (\Omega - M)|x| - h\cdot\left[q + A\left(\frac{1}{h}(|x| + x)\right)\right]; \tag{3}$$

(b) *If x satisfies the implicit fixed-point Eq. (3), then*

$$u = \frac{1}{h}(|x| + x), \quad v = \frac{1}{h}\Omega(|x| - x) \tag{4}$$

is a solution of the nonlinear complementarity Problem (1).

Proof. We first demonstrate the validity of (a). As u is a solution of the nonlinear complementarity Problem (1). It is a nonnegative vector and can be expressed in the form

$$u = \frac{1}{h}(|x| + x)(\forall x \in R^n).$$

Define another nonnegative vector

$$v = \frac{1}{h}\Omega(|x| - x).$$

Then we have $u^T v = 0$ and $v = Mu + q + A(u)$ if and only if

$$(\Omega + M)\cdot x = (\Omega - M)\cdot|x| - h\cdot\left[q + A\left(\frac{1}{h}\cdot(|x| + x)\right)\right].$$

By utilizing the matrix splitting $M = F - G$,

$$(\Omega + F - G)\cdot x = (\Omega - M)\cdot|x| - h\cdot\left[q + A\left(\frac{1}{h}\cdot(|x| + x)\right)\right],$$

$$(\Omega + F)\cdot x = G\cdot x + (\Omega - M)\cdot|x| - h\cdot\left[q + A\left(\frac{1}{h}\cdot(|x| + x)\right)\right]. \tag{5}$$

Therefore, we obtain the implicit fixed-point Eq. (3).