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Complex dynamics analysis for a duopoly Stackelberg game model with bounded rationality

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ABSTRACT

In view of the effect of differences between plan products and actual products, a duopoly Stackelberg model of competition on output is formulated. The firms announce plan products sequentially in planning phase and act simultaneously in production phase. Backward induction is used to solve subgame Nash equilibrium. The equilibrium outputs and equilibrium profits are affected by cost coefficients. For the duopoly Stackelberg model, a nonlinear dynamical system which describes the time evolution with bounded rationality is analyzed. The equilibria of the corresponding discrete dynamical systems are investigated. The local stability analysis has been carried out. The stability of Nash equilibrium gives rise to complex dynamics as some parameters of the model are varied. Numerical simulations were used to show bifurcation diagram, stability region and chaos. It is also shown that the state variables feedback and parameter variation method can be used to keep the system from instability and chaos.

1. Introduction

An oligopoly is a market form between monopoly and perfect competition, in which a market has a dominant influence on a small number of firms (oligopolists) [1]. The dynamic of an oligopoly game is more complex because firms must consider not only the behaviors of the consumers, but also the reactions of the competitors. The first formal theory of oligopoly was introduced by Cournot, in 1838 [2]. Significant additions to the theory were made exactly one hundred years later by H. von Stackelberg [3]. In the repeated oligopoly game all players maximize their profits. Recently, the dynamics of the duopoly game has been studied [4–16]. The general formula of the oligopoly model with a form of bounded rationality has been investigated [6]. The results show that the dynamics of the game can lead to complex behaviors such as cycles and chaos. The complex dynamics of a bounded rationality duopoly game with a nonlinear demand function has been studied [10]. Depending on the strategy that the firms use and the expectations of the output the firms have to maximize, the modification of the duopoly game has been discussed [11,12]. With bounded rationality, Ref. [13] examined the dynamical behavior of Bowley's model. Furthermore, Ref. [14] used the Jury condition to discuss the stability of a modification of Puu's model. The development of complex oligopoly dynamics theory has been reviewed in Ref. [16]. Other studies on the dynamics of oligopoly models with more firms and other modifications have been studied in Ref. [17–21]. Based on bounded rationality, a linear dynamic system for the duopoly game of renewable resource extraction was proposed [22]. Also in the past decade, there has been a great deal of interest in chaos control of duopoly games because of its complexity in Ref. [23] and Ref. [24] and their references.

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Most of the previous works are based on the Cournot model and the modifications to discuss the complex dynamics. There is little literature dealing with Stackelberg model and the modifications in studying dynamical behaviors with bounded rationality. The Stackelberg games are natural models for many important applications that involve human interaction. Existing algorithms for Stackelberg games find optimal solutions (leader strategy) efficiently, but they critically assume that the follower plays optimally. Unfortunately, in many applications, players facing human followers (adversaries) may deviate from their expected responses to the game theoretic optimal choice, because of their bounded rationality and limited observation [25]. Thus, a human adversary may cause an unacceptable degradation in the leader's reward [26].

Expectation plays an important role in modelling economic phenomena. A firm can choose its expectation rules of many available techniques to adjust its strategy. The present work aims to formulate a duopoly Stackelberg game with bounded rationality and study the dynamical behaviors. The leader firm chooses strategic variable first, then the follower firm chooses strategic variable. In the subsequent stages, two firms update their strategies in order to maximize their profits in the market. Each firm adjusts its strategy according to the expected marginal profit, therefore the decision of each firm depends on local information about its output. In this Stackelberg game each firm tries to maximize its profit according to local information of its strategy.

This paper is organized as follows. In Section 2, the Stackelberg game and the duopoly game with bounded rationality are briefly described. Some properties about the equilibrium output and equilibrium profit are investigated. The dynamics for a duopoly Stackelberg game model with bounded rationality are analyzed. The local stability analysis has been carried out. In Section 3, we present the numerical simulations to verify our theoretical results. In Section 4, we exerted control on the duopoly Stackelberg game model. Finally, some remarks are presented in Section 5.

2. The model

2.1. The Stackelberg game and duopoly Stackelberg model

The original Stackelberg model is a sequential quantity choice game in a homogeneous product market. In Stackelberg games, one player, the leader, commits to a strategy publicly before the remaining players, the followers, make their decisions. The followers selfishly optimize their rewards, considering the action chosen by the leader. The leader knows ex ante that the followers observe his action. The decision and actual performance of the followers will influence the cost and benefit of the leader, so the followers must make decision to predict the decision of the leader, that is to say, the decision of the leader and followers is influenced mutually. A Stackelberg equilibrium is a subgame perfect equilibrium of the above game.

The classic Stackelberg game is divided into two stages. In stage 1, the*planning phase*, each player chooses strategies, and concludes forward contracts for output. In stage 2, the *production phase*, they choose the quantities to be produced. The players act sequentially in planning phase, act simultaneously in production phase, and the choices made in stage 1 are common knowledge in stage 2. There are no costs of production besides the costs of capacity. The forward sales are priced competitively in that the eventual resulting market price is anticipated correctly in equilibrium.

We consider two firms, labelled by $i = 1, 2,$ producing the same goods for sale in the market. Firm 1 is the Stackelberg leader and firm 2 is the follower. Production decisions of both firms are made at discrete periods $t = 0, 1, 2, \ldots$. Let $q_i(t) > 0$ represents the output of firm i during period t, with a production cost function $C_i(q_i)$. The price prevailing in period t is determined by the total supply $q(t) = q_1(t) + q_2(t)$ through a demand function $p = f(q)$. In this model the demand function is assumed linear, which has the form $f(q) = a - bq,$ where a and b are positive constants.

Let Q_i be the announced plan products of the firm $i, i = 1, 2$ respectively. There is a difference between the announced product and the actual output of firm i during period t. In this work we assume that the firms use different production method and the cost function is proposed in the nonlinear form

$$C_i(q_i) = c_i(q_i - Q_i)^2, \quad i = 1, 2$$

where the parameters c_i are positive shift parameters to the cost function of the firm i, $i = 1, 2$ respectively. In fact, there are many factors affecting the difference between ideal product and actual product. For the sake of analysis, all of these factors are summarized as the cost coefficient c_i. With these assumptions, the single-period profit of the firm i is given by

$$\Pi_i(q_1, q_2) = q_i(a - bq) - c_i(q_i - Q_i)^2, \quad i = 1, 2$$

The empirical estimate of marginal profit for the firm i at the point (q_1, q_2) is given by

$$\frac{\partial \Pi_i}{\partial q_i} = a + 2c_iQ_i - 2(b + c_i)q_i - bq_j, \quad i = 1, 2, \ i \neq j \tag{1}$$

In order to maximize profit for the firm i, let the partial derivative of Π_i respect to q_i equal to zero.

$$\frac{\partial \Pi_i}{\partial q_i} = a + 2c_iQ_i - 2(b + c_i)q_i - bq_j = 0, \quad i = 1, 2, \ i \neq j$$