Applied Mathematics and Computation 271 (2015) 489–501

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Computation of a multi-choice goal programming problem

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A R T I C L E I N F O

Keywords:
Multi-criteria decision making
Multi-choice goal programming
Multiple aspiration levels

A B S T R A C T

The standard goal programming problem allows decision maker to assign an aspiration level to an objective function. In real life decision making problems, the decision maker always seeks for suitable aspiration level i.e. “the more suitable the better”. Therefore, a decision maker is allowed to assign multiple number of aspiration levels to an objective function. The aim of the decision maker is to select an appropriate aspiration level for an objective function that minimizes the deviations between the achievement of goal and the aspiration levels. The traditional goal programming techniques cannot be used for solving such type of multi-choice goal programming problem. This paper presents an equivalent model of the multi-choice goal programming problem by using Vandermonde’s interpolating polynomial, binary variables and least square approximation method. The equivalent model is solved by existing method/software. Two illustrative examples are presented in support of the proposed methodology.

1. Introduction

The term ‘decision’ carries different meanings, depending upon the nature of decision maker. A decision maker may be a lawyer, a businessman, a psychologist or a statistical or a general person. It might be behavioral action, mathematical model, or a specific kind of information processing. It is very difficult to represent decision making problems in mathematical models due to conflicts of resources and incompleteness of available information. In realistic situations decision making problems require to consider multiple objectives on one hand and various types of uncertainties on the other hand. There exists different methods to handle different uncertainties.

Several techniques, namely utility function approach, goal programming approach, reference point method, interactive approach etc. exist for the solution of multi-criteria decision making problems. Among them the most popular is goal programming (GP) approach. GP is an analytic approach devised to address decision making problems where targets have been assigned to all the attributes. The decision maker (DM) is interested in minimizing the non-achievement of the goals. GP was first addressed by Charnes and Cooper [1]. Since the mid 70*s*, due to the seminal works by Lee [2] and Ignizio [3] an impressive revolution of GP applications and theoretical developments took place. Now-a-days, GP is the key-technique to work with multi-criteria decision making problems. Lee [2] and Ignizio [4] wrote impressive books on GP. Tamiz et al. [5] provided an up to date review on GP. The

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http://dx.doi.org/10.1016/j.amc.2015.09.030

contributions of Tamiz et al. [6], Romero et al. [7], Ijiri [8], Schniederjans [9], Zeleny and Cochrane [10] were remarkable for the development of GP.

The distinction between various types of generalized goal programming, is made on the basis as how one actually measures the "goodness" of any solution (the value of X) to the set of goals. This is a typical method facilitated by means of the concept of "goal deviations" and the "achievement function". By the philosophy of goal programming problem DM chooses a target value and decides whether to penalize positive or negative deviations from the target. However, DMs are not interested in the specific fixed deterministic targets associated with certain attributes in real-life.

DMs prefer flexibility and suitability. It is observed that DMs are interested in a set of deterministic targets associated with an attribute. Keeping this in mind, Chang [11] proposed a new idea for programming the multi-choice aspiration level problem and named it as Multi-Choice Goal Programming (MCGP) Problem. He introduced the multiplicative terms of binary variables in order to tackle with multi-choice aspiration levels associated with each goal. The way he introduced the multiplicative terms of binary variables is too difficult to implement and is not easily understood by industrial participants. In Chang [12], although multiple aspiration levels are assigned to a goal, it is difficult to describe role of all aspiration levels in that goal. It replaces multiplicative terms of the binary variables by taking the help of a continuous variable, with a range of interval values as the lower and upper bound of each objective function. A new concept of constrained multi-choice goal programming is introduced for constructing the relationships between goals. This paper presents a new approach to search the appropriate set of aspiration levels from multiple sets of aspiration levels using multiplicative terms of binary variables.

Biswal and Acharya [13,14], Acharya and Biswal [15] used binary variables in order to transform a multi-choice linear programming problem to an equivalent mathematical model. Using the concept of Chang [11], Liao [16] formulated multi-segment goal programming. Acharya and Acharya [17] generalized the transformation technique proposed by Biswal and Acharya [14]. Biswal and Acharya [18] used interpolating polynomial approach to solve multi-choice linear programming problem. After using the interpolation, the formulated mathematical model was a mixed integer nonlinear programming problem. Chang et al. [19] used his own technique to select a suitable house for homebuyer. Ustun [20] used conic scalarization function for formulating the MCGP. Fuzzy multi-choice goal programming problem was first addressed by Bankian-Tabrizi et al. [21]. Chang et al. [22] used multi-coefficients goal programming for group pricing problem. Multi choice mixed integer goal programming problem was carried out by Da Silva et al. [23]. They mainly focused on decisions on the choice of production process, including storage stages and distribution.

The paper is organized as follows: Section 2 contains basic preliminaries followed by "mathematical model" in Section 3. In Section 4 equivalent models for the proposed MCGP are presented. Two numerical examples are provided in Section 5 to justify the methodology. In Section 6, results and discussions are presented. Finally concluding remarks are made following supporting references.

2. Basic preliminaries

The aim of the goal programming is to minimize the deviations between the achievement of goals and their aspiration levels. According to the philosophy of satisficing we are interested in measuring the non-achievement of each goal. This is the unwanted deviations from the aspiration levels (i. e. the value of each goal 'g'). We let

$$d_i = \textit{the deviation between 'the aspiration level' and 'the acheivement of goal'}.$$
$$\text{'or'} d_i = g_i - f_i(X).$$

Where g_i is the aspiration level and $f_i(X)$ is the objective, which is to be achieved.

Hence, we can express the general goal programming as:

$$\begin{aligned} &\min\ :\ |f_i(X) - g_i| \text{ for } i = 1, 2, \ldots, m \\ &\text{subject to} \\ &\qquad X \in \mathbb{R} \end{aligned}$$

where $\mathbb{R}$ is a feasible set.

The three oldest and still most widely used forms of GP are used to minimize the unwanted deviations. The methods are as follows:

1. Lexicographic Goal Programming (LGP), also known as non-Archimedean or preemptive GP.
2. Weighted GP (WGP), also known as Archimedean GP.
3. Min–Max GP (MGP) also known as Chebyshev or Fuzzy Programming.

The mathematical formulations for LGP is expressed as follows:

$$\begin{aligned} &\text{lexicographically min} : \bar{a} = (a_1, a_2, a_3, \ldots, a_k, \ldots, a_K) \\ &\qquad \text{subject to} \\ &\qquad\qquad f_i(X) + \eta_i - \beta_i = b_i \qquad \forall i \\ &\qquad\qquad X, \bar{\eta}, \bar{\rho} \geq 0 \end{aligned}$$