Applied Mathematics and Computation 271 (2015) 757–767

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Chaotic sliding mode controllers for uncertain time-delay chaotic systems with input nonlinearity

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ARTICLE INFO

Keywords:
Chaos control
Sliding mode control (SMC)
Time-delay
Input nonlinearity
Lyapunov theory
Linear matrix inequality (LMI)

ABSTRACT

This paper presents a novel robust chaotic controller for stabilizing uncertain time-delay chaotic systems with input nonlinearity. Based on dynamic output sliding mode control (SMC), Lyapunov theory and linear matrix inequality (LMI) technique, the stability of overall uncertain time-delay chaotic system with input nonlinearity under the proposed control scheme is guaranteed without the state predictor. The selection of sliding surface and the design of control law are two important issues, which have been addressed. A priori knowledge of upper bounds of uncertainties is not required. Furthermore, the proposed scheme ensures robustness against input nonlinearity, time-delays, nonlinear real-valued functions, parameter uncertainties and disturbances simultaneously. Finally, simulation results demonstrate the validity of the proposed control methodology.

1. Introduction

Chaos is an interesting and common phenomenon in nature. Chaotic systems are characterized by prominent features such as broad Fourier transform spectra, fractal properties of the motion in phase space and extraordinary sensitivity to initial conditions and system parameter variations. The sensitivity to initial conditions is popularly known as the butterfly effect [1]. An interesting feature of chaotic systems is that the change of the parameter values can obtain different dynamical behaviors of chaotic systems. For example, the unified chaotic system [2] becomes Lorenz system, critical system and Chen system when the system parameter $\alpha = 0$, $\alpha = 0.8$ and $\alpha = 1$, respectively. As the system parameter α changes continuously from 0 to 1, the resulting system is always chaotic.

On the other hand, bifurcation phenomena [3] can be occurred in complex chaotic systems when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden qualitative or topological change in its behavior. In particular, period-doubling bifurcation is a typical route to chaos in many nonlinear dynamical systems. In [4], an energy balance model (EBM) is developed for analyzing bifurcation and chaos phenomena of capacitor energy and output voltage when the converter parameter is varying. It is found that the capacitor energy and output voltage dynamic behaviors exhibit the typical period-doubling route to chaos by increasing the feedback gain constant of proportional controller. In many applications chaotic response is undesirable, because even small perturbations may cause trajectories to diverge exponentially [5]. Hence, it should be avoided or totally suppressed in practice.

Since chaos control problem was firstly considered by [6], the stabilization of chaotic systems has been paid much attention and various control strategies have been applied to realize chaos control and synchronization such as adaptive control [7–15],

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http://dx.doi.org/10.1016/j.amc.2015.09.058

sliding mode control [2,16–30], fuzzy control [31,32], linear feedback control [33,34], polynomial approach [35] and harmonic approach [36–39]. In addition, several design methods [40–45] for the stabilization of systems with uncertainties have been investigated. However, these papers assumed that all states are known. In practice, full measurement of the state vector is not feasible. In many systems, only the system output is available.

Recently, in [46], an adaptive output feedback control method is proposed for a class of uncertain chaotic systems. In [47], an adaptive output feedback method is presented for stabilizing a class of uncertain nonlinear systems. However, these methods cannot deal with the robustness against input nonlinearity and time-delays. In [48], a fuzzy output feedback control is developed for uncertain nonlinear systems with nonsymmetric dead-zone input. However, this method cannot deal with the effects caused by time-delays. The problem of input nonlinearity and time-delays is often encountered in various engineering systems, and the existence of input nonlinearity and time-delays frequently becomes a source of instability or performance deterioration of systems. The stabilization of chaotic systems which have input nonlinearity and time-delays is more difficult than that of systems without input nonlinearity and time-delays. To the best of the author's knowledge, there is little work undertaken on the stabilization problem of uncertain chaotic systems with input nonlinearity, state delays, input delays, nonlinear real-valued functions and unknown parameter uncertainties by using partial known state information, which is still open in the literature. This motivates the current research.

In addition, SMC method is an effective robust control approach for uncertain systems. SMC method has significant attractive features such as fast response, good transient performance, insensitiveness to the matching parameter uncertainties and external disturbances [49,50]. In this paper, a dynamic output SMC method is developed for a class of uncertain chaotic systems with immeasurable states, input nonlinearity, state delays, input delays, nonlinear real-valued functions and unknown parameter uncertainties. A state estimator is constructed to estimate a full set of states. Based on Lyapunov theory and LMI technique, the stability of overall closed-loop uncertain time-delay chaotic system with input nonlinearity is guaranteed. The main contribution of this paper is as follows: (1) the proposed method does not require a state transformation or a state predictor to convert the original system into a delay-free system. The full measurement of states and a priori knowledge of upper bounds of uncertainties are not required. (2) The design technique is simple and computationally efficient. The state observer and control law are constructed from the positive-definite solutions of a LMI instead of two LMIs, which provides a significant advantage over the conventional state observer control scheme. (3) The order of the motion equation in the sliding mode is equal to the order of the original system, rather than reduced by the number of dimension of control inputs. The robustness of the system can be guaranteed throughout the entire response of the system starting from the initial time instance. (4) It can be easily extended to the case of multiple state and input delays.

This paper is organized as follows. Section 2 briefly states the problem formulation and assumptions. Section 3 provides the proposed dynamic output SMC scheme. The selection of sliding surfaces, the design of a dynamic output sliding mode controller, and the stability of overall closed-loop uncertain time-delay chaotic systems with input nonlinearity are addressed. Section 4 provides results from numerical simulations, and verifies the efficacy of the proposed scheme. Finally, a conclusion is provided in Section 5.

2. Problem formulation and assumptions

In this paper, a class of uncertain time-delay chaotic systems with input nonlinearity is described as:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-d) + (B_h + \Delta B_h(t))u(t-h) + B(\Phi(u(t) + f(x) + w(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R$ is the control input, $\Phi(u(t)) \in R$ denotes a nonlinear vector function containing sector nonlinearity, $y(t) \in R^p$ is the output, $f(x)$ is a nonlinear real-valued function vector, and $w(t)$ is the external disturbance. A, A_d, B, B_h and C are known constant matrices of appropriate dimensions, $\Delta A(t)$, $\Delta A_d(t)$, and $\Delta B_h(t)$ are unknown time-varying parameter uncertainties. $d > 0$ and $h > 0$ represent known constant time delays in state and input respectively.

The following assumptions are necessary for further development.

Assumption 1. The pair (A, B) is controllable and the pair (A, C) is observable.

Assumption 2. $\text{rank}(CB) = 1$; i.e. $\text{rank}(B) = 1$

Assumption 3. There exist unknown time-varying matrix functions of appropriate dimension $D_A(t)$, $D_d(t)$ and $D_h(t)$ such that $\Delta A(t) = BD_A(t)$, $\Delta A_d(t) = BD_d(t)$ and $\Delta B_h(t) = BD_h(t)$.

Assumption 4. The nonlinear input function $\Phi(u(t))$ is a continuous nonlinear vector function containing sector nonlinearity and assumed to satisfy

$$\beta_2 u^2(t) \geq u(t)\Phi(u(t)) \geq \beta_1 u^2(t), \quad \Phi(0) = 0 \tag{2}$$

where β_1 and β_2 are nonzero positive constants.

Remark 1. In Assumption 3, the plant uncertainties $\Delta A(t)$, $\Delta A_d(t)$ and $\Delta B_h(t)$ lie in the image of input matrix B, i.e. $\Delta A(t) = BD_A(t)$, $\Delta A_d(t) = BD_d(t)$ and $\Delta B_h(t) = BD_h(t)$. This condition is so-called matching condition [49,51], which means