Applied Mathematics and Computation 271 (2015) 790–794

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On certain topological indices of the line graph of subdivision graphs

Muhammad Faisal Nadeem [a], Sohail Zafar [b,*], Zohaib Zahid [b]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Lahore, Pakistan
[b] University of Management and Technology, Lahore, Pakistan

ARTICLE INFO

Keywords:
ABC_4 index
GA_5 index
Line graph
Subdivision graph

ABSTRACT

In QSAR/QSPR study, topological indices such as Shultz index, generalized Randic index, Zagreb index, general sum-connectivity index, atom-bond connectivity (ABC) index and geometric-arithmetic (GA) index are utilized to guess the bioactivity of chemical compounds. A topological index in fact relates a chemical structure with a numeric number. Graph theory has established a significant use in this area of research. In this paper we computed ABC_4 and GA_5 indices of the line graph of tadpole, wheel and ladder graphs using the notion of subdivision.

© 2015 Elsevier Inc. All rights reserved.

1. Introduction and preliminaries

In this article we will consider only simple graphs without loop and multiple edges. Let G be a simple graph, with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of edges that are incident to it and $S_u = \sum_{v \in N_u} d_v$ where $N_u = \{v \in V(G) | uv \in E(G)\}$. N_u is also known as the set of neighbor vertices of u. For any natural number d, we define $V_d = \{u \in V(G) \mid S_u = d\}$.

Topological indices are the numerical quantities which represent the structure of any simple finite graph. They are invariant under the graph isomorphisms. The importance of topologically indices is generally linked with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) (see [18]).

The first degree-based connectivity index for graphs developed by using vertex degrees is Randic index [15]. The Randic index of a graph G is defined as

$$R(G) = \sum_{uv \in E(G)} (d_u d_v)^{-1/2}.$$

Later, this index was generalized for any real number α, and known as the generalized Randic index $R_\alpha(G)$:

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^{\alpha}.$$

In 2010, general sum-connectivity index $\chi_\alpha(G)$ has been introduced in [26]:

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_u + d_v)^{\alpha}.$$

* Corresponding author. Tel.: +92 3214242349.
E-mail addresses: mfaisalnadeem@ymail.com (M.F. Nadeem), sohailahmad04@gmail.com (S. Zafar), zohaib_zahid@hotmail.com (Z. Zahid).

http://dx.doi.org/10.1016/j.amc.2015.09.061
0096-3003/© 2015 Elsevier Inc. All rights reserved.

The atom–bond connectivity (ABC) index, introduced by Estrada et al. in [3]. The *ABC* index of graph *G* is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}.$$

Vukicevic and Furtula introduced the geometric–arithmetic (GA) index in [24]. The *GA* index for graph *G* is defined by

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}.$$

The fourth member of the class of ABC index was introduced by Ghorbani and Hosseinzadeh [6] as:

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} \tag{1}$$

The fifth GA index was introduced by Graovac et al. in [7] as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \tag{2}$$

For a collection of recent results on degree-based topological indices, we refer the interested reader to the articles [1,2,4,5, 8–14,19,21–23,25].

Now we define some notations of the graph theory. The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2. The line graph $L(G)$ of graph G is the graph whose vertices are the edges of G, two vertices e and f are incident if and only if they have a common end vertex in G. The tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of n vertices with a path of length k. A ladder L_n is obtain by taking Cartesian product of two paths $P_n \times P_2$. A wheel graph W_n of order n composed of a vertex, which will be called the hub, adjacent to all vertices of a cycle of order n i.e., $W_n = C_n + K_1$. The cycle will be called the rim of the wheel, and the edges connecting the hub to the vertices of the rim will be called the spokes.

2. Topological indices of $L(S(G))$

In 2011, Ranjini et al. calculated the explicit expressions for the Shultz index of the subdivision graphs of the tadpole, wheel, helm and ladder graphs [17]. They also studied the Zagreb indices of the line graph of the tadpole, wheel and ladder graphs with subdivision in [16]. In 2015, Su and Xu calculate the general sum-connectivity index and co-index of the $L(S(T_{n,k}))$, $L(S(W_n))$ and $L(S(L_n))$ in [20].

Motivated by the results of [20], we computed ABC_4 and GA_5 indices of the line graph of the tadpole, wheel and ladder graphs using the notion of subdivision.

2.1. ABC_4 and GA_5 indices of $L(S(T_{n,k}))$

Theorem 2.1. *Let G be the line graph of subdivision graph of the tadpole graph. Then ABC_4 index is given by*

$$ABC_4(G) = \begin{cases} \frac{\sqrt{14}}{8} + \frac{2\sqrt{42}}{21} + \frac{\sqrt{182}}{14} + \frac{\sqrt{110}}{10} + \frac{(2n-5)\sqrt{6}}{4} + \frac{\sqrt{35}}{5}, & \text{if } k = 1; \\ \frac{\sqrt{2}}{2} + \frac{\sqrt{10}}{5} + \frac{3\sqrt{110}}{20} + \frac{3\sqrt{14}}{8} + \frac{(2n-5)\sqrt{6}}{4} + \frac{\sqrt{35}}{5}, & \text{if } k = 2; \\ \frac{\sqrt{2}}{2} + \frac{\sqrt{15}}{6} + \frac{3\sqrt{110}}{20} + \frac{3\sqrt{14}}{8} + \frac{(n+k-5)\sqrt{6}}{2} + \frac{3\sqrt{35}}{10}, & \text{if } k > 2. \end{cases}$$

Proof. First consider the line graph $L(S(T_{n,k}))$ with $k > 2$. In this graph there are total $2(n+k)$ vertices, consisting 3 vertices of degree 3, one vertex of degree 1 and remaining all the vertices of degree 2. Therefore $|V_2| = 1, |V_3| = 1, |V_5| = 3, |V_8| = 3$ and $|V_4| = 2(n+k-4)$ as shown in Fig. 1(b). Hence, we get the edge partition, based on the degree sum of neighbor vertices of each vertex as shown in Table 3. We apply Formula (1) to Table 3 and obtain the required result. By similar arguments we can obtain the expressions of $ABC_4(G)$ index for $k = 1$ and 2 by viewing edge partitions in Tables 1 and 2, respectively. □

Table 1
The edge partition of $L(S(T_{n,k}))$ for $k = 1$.

(S_u, S_v) where $uv \in E(G)$	(3,7)	(5,8)	(7,8)	(8,8)	(4,4)	(4,5)
Number of edges	1	2	2	1	$2n-5$	2

Download English Version:

https://daneshyari.com/en/article/4626200

Download Persian Version:

https://daneshyari.com/article/4626200

Daneshyari.com