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Exponential stability and instability of impulsive stochastic functional differential equations with Markovian switching

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ABSTRACT

In this paper, based on the Lyapunov second method and Razumikin techniques, we establish some novel criteria on pth moment exponential stability, almost exponential stability and instability of impulsive stochastic functional differential equations (ISFDEs) with Markovian switching. The findings show that impulsive stochastic functional equations with Markovian switching can be exponentially stabilized by impulses. Finally, an example is presented to illustrate the effectiveness and efficiency of the obtained results.

1. Introduction

In the past decades, impulsive differential equations have been used efficiently in modeling many practical problems that arise in science and engineering [1–3]. Impulsive control is of great importance in science and industry. Wang and Liu [4] considered impulsive stabilization of delay differential system via the Lyapunov–Razumikhin method. The authors in [5] studied global exponential stability of impulsive differential equations with any time delays. Hespanha et al. [6] obtained Lyapunov conditions for input-to-state stability of impulsive systems.

Besides impulsive effects, external stochastic perturbations also exist in many real systems, involving such fields as medicine and biology, economics, mechanics, electronics, telecommunications, etc., [7–12]. In 1984, Chang [7] discussed Razumikhin-type uniformly asymptotic stability of stochastic functional differential equations (SFDEs) with finite delay. Mao [7,8] further obtained some Razumikhin-type theorems on pth moment exponential stability and almost exponential stability of SFDEs. Jankovic et al. [12] discussed both pth moment and almost exponential stability of solutions to neutral SFDEs and neutral stochastic differential equations (SDEs). Recently, stability of impulsive stochastic differential equations (ISDEs) and ISFDEs have attracted more and more attention [13–25]. Liu [13] has derived comparison principles of existence and uniqueness and stability of solutions for ISDEs. Alwan et al. [14] considered the existence, continuation, and uniqueness problems of stochastic impulsive systems with time delay. By using the properties of M-cone and stochastic analysis technique, Yang et al. [18] investigated exponential p-stability of impulsive stochastic delays differential systems. Based on the fixed point theory, some results on the existence of mild solutions for a class of impulsive neural stochastic functional integro-differential inclusions with nonlocal initial conditions and resolvent operators were derived in [19]. Sakthivel et al. studied the existence and asymptotic stability in pth moment of mild solutions to nonlinear impulsive stochastic differential equation with infinite delay in [20]. Xu [21] considered mean square exponential stability for a class of linear impulsive SDEs by employing the formula for the variation of parameters. In addition,

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Razumikhin-type stability theorems for ISFDEs were established in [16,22,23]. The theory of ISDEs has applications to chaos control, chaos synchronization and neural networks, see [26,27] and references therein.

Markovian jump systems, as a special kind of SDEs introduced by Krasovskii and Lidskii [28] in 1961, have received increasing interests, see [29–32] and references therein. More recently, some results on impulsive stochastic functional equations with Markovian switching have been reported [33–38]. Wu and Sun [33] have presented some stability criteria of pth moment stability for stochastic differential equations with impulsive jumper and Markovian switching, without considering time delay. Peng and Zhang [34] derived some new criteria on pth moment stability of stochastic functional differential equations with Markovian switching. Kao et al. obtained some results on delay-dependent exponential stability of impulsive Markovian jumping Cohen–Grossberg neural networks with reaction–diffusion and mixed delays in [35]. Li and Kao [36] discussed the stability problem of stochastic reaction–diffusion systems with Markovian switching and impulsive perturbations. Wu et al. [37] have considered pth moment stability of impulsive stochastic delay differential systems with Markovian switching. Zhu [38] has derived some new criteria on pth moment exponential stability of impulsive stochastic functional differential equations with Markovian switching. However, to the best of our knowledge, little work has been done on almost exponential stability and instability of impulsive stochastic functional differential equations with Markovian switching, which is still an open problem and remains challenging.

Motivated by the above discussion, in this paper, we aim to use the Razumikhin techniques and Lyapunov functions to investigate the exponential stability and instability of ISFDEs with Markovian switching. Section 2 is preliminaries. Some definitions are presented. Section 3 is the main results. Based on the Lyapunov second method, Gronwall inequality and Razumikin techniques, we establish some novel criteria on pth moment exponential stability, almost exponential stability and instability of impulsive stochastic functional differential equations (ISFDEs) with Markovian switching. The stability analysis is much more complicated because of the existence of impulsive effects and stochastic effects at the same time. Moreover, our results show that impulses make contribution to the exponential stability of stochastic differential systems with any time delay even they are unstable. An example is provided in Section 4. Section 5 concludes the paper.

Notations: Throughout this paper, let $(\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathbb{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration is increasing and right continuous while $\mathbb{F}_0$ contains all $\mathbb{P}$-null sets). $B(t) = (B_1(t), \dots, B_d(t))^T$ is a d-dimensional Brownian motion defined on the probability space. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers and $\mathbb{R}^n$ the n-dimensional real space equipped with Euclidean norm $|\cdot|$. I stands for the identity matrix of appropriate dimensions. $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a matrix. If A is a vector or matrix, its transpose is A^T, and its norm $\|A\| = \sqrt{\lambda_{\max}(AA^T)}$. N is the set of positive integers, i.e. $N = \{1, 2, \dots\}$. Let $\tau > 0$ and $PC([-\tau, 0]; \mathbb{R}^n) = \{\phi : [-\tau, 0] \to \mathbb{R}^n | \phi(t^+) = \phi(t)$ for all $t \in [-\tau, 0), \phi(t^-)$ exists and $\phi(t^-) = \phi(t)$ for all but at most a finite number of points $t \in (-\tau, 0]\}$ be with the norm $|\phi| = \sup_{-\tau\leq\theta\leq 0} \mathbb{E}|\phi(\theta)|$ where $\phi(t^+)$ and $\phi(t^-)$ denote the right-hand limits of function $\phi(t)$ at t, respectively. For $p > 0$ and $t \geq 0$, let $PC^p_{F_t}([-\tau, 0]; \mathbb{R}^n)$ denote the family of all $\mathfrak{S}_t$-measurable $PC([-\tau, 0]; \mathbb{R}^n)$ valued random processes $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$, such that $\sup_{-\tau\leq\theta\leq 0} \mathbb{E}|\phi(\theta)|^p < \infty$, where $\mathbb{E}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$. And $PC^p_{\mathbb{F}_t}(\Omega; \mathbb{R}^n)$ denote the family of all $\mathbb{F}_t$ measure $\mathbb{R}^n$-valued random variables X, such that $E|X|^p < \infty$. Let $PC^b([-\tau, 0]; \mathbb{R}^n)$ be the family of all bounded $PC([-\tau, 0]; \mathbb{R}^n)$-valued functions.

2. Preliminaries

Consider the impulsive stochastic functional differential equations (ISFDEs) with Markovian switching as follows:

$$\begin{cases} dx(t) = f(x_t, t, r(t))dt + g(x_t, t, r(t))dB(t), & t \neq t_k, t \geq t_0 \\ \Delta x(t_k) = I_k(x(t_k^-), t_k), & k \in N \\ x_{t_0} = \phi(\theta), & -\tau \leq \theta \leq 0 \end{cases} \tag{1}$$

where the initial value $\phi \in PC^b([-\tau, 0]; \mathbb{R}^n), x(t) = (x_1(t), \dots, x_n(t))^T, x_t = x(t+\theta) \in PC^p_{F_t}([-\tau, 0]; \mathbb{R}^n), f : PC^p_{F_t}([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}_+ \times \mathbb{S} \to \mathbb{R}^n, g : PC^p_{F_t}([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}_+ \times \mathbb{S} \to \mathbb{R}^{n\times d}, I_k(x(t_k^-), t_k) : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ represents the impulsive perturbation of x at time t_k. The fixed moments of impulse times t_k satisfy $0 \leq t_0 < t_1 < \cdots < t_k < \cdots, t_k \to \infty (as\ k \to \infty), \Delta x(t_k) = x(t_k) - x(t_k^-)$. $r(t)(t \geq 0)$ is a right-continuous Markov chain on a probability space $(\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t\geq 0}, \mathbb{P})$ taking values in a finite state space $\mathbb{S} = \{1, 2, \dots, \mathrm{N}\}$ with generator $\Pi = \{\pi_{ij}\}$ given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases} \tag{2}$$

where $\Delta > 0$, $\lim_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$, and $\pi_{ij} \geq 0$, $(i, j \in \mathbb{S}, j \neq i)$, is the transition rate from mode i at time t to mode j at time $t + \Delta$, and $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$, for each $i \in \mathbb{S}$.

As usual, we assume that $f(\phi, t, r(t))$ and $g(\phi, t, r(t))$ are continuous for almost all $t \in [t_0, \infty)$ and are Lipschitz in ϕ in each compact set in $PC^p_{F_t}([-\tau, 0]; \mathbb{R}^n)$; $I_k(x, t_k)$ is Lipschitz in x in each bounded set in $\mathbb{R}^n$, $k \in N$. For any $\phi \in PC^b([-\tau, 0]; \mathbb{R}^n)$, there exists a unique stochastic process satisfying Eq. (1) denote by $x(t; t_0; \phi)$, which is continuous on the right-hand side and limitable on the left-hand side. Moreover, we assume that $f(0, t, r(t)) \equiv 0$, $g(0, t, r(t)) \equiv 0$, and $I_k(0) \equiv 0$ for all $t \geq t_0$, $k \in N$, then Eq. (1) admits a trivial solution $x(t) \equiv 0$.

Definition 1. The trivial solution of Eq. (1) or, simply, system (1) is said to be