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Consensus rate regulation for general linear multi-agent systems under directed topology

Tao Feng [a], Huaguang Zhang [a,b,*], Yanhong Luo [a], Hongjing Liang [a]

[a] College of Information Science and Engineering, Northeastern University, 110819 Shenyang, LN, PR China
[b] State Key Laboratory of Synthetical Automation for Process Industries (Northeastern University), 110004 Shenyang, LN, PR China

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ABSTRACT

Recently, optimization in distributed multi-agent coordination has been studied concerning convergence speed. The optimal convergence speed of consensus for multi-agent systems consisting of general linear node dynamics is still an open problem. This paper aims to design optimal distributed consensus protocols for general identical linear continuous time cooperative systems which not only minimize some local quadric performances, but also regulate the consensus rate (including convergence rate and damping rate) for the multi-agent systems. The graph topology is assumed to be fixed and directed. The inverse optimal design method is utilized and the resulting optimal distributed protocols place part of close-loop poles of the global disagreement systems at specified locations asymptotically, while the remains far from the imaginary axis enough. It turns out that for the identical linear continuous time multi-agent systems, the convergence speed has no upper bound. The main advantages of the developed method over the LQR design method are that the resulting multi-agent systems can achieve specified consensus rate asymptotically and the resulting protocols have the whole right half complex plane as its asymptotical consensus region. Numerical examples are given to illustrate the effectiveness of the proposed design procedures.

1. Introduction

Cooperative distributed control of multi-agent systems in accomplishing a common task has received increasing demands and been a priority research subject for a variety of military and civilian applications. All agents can communicate with each other and exchange information such as their relative positions, target, etc. and use the data to develop distributed but coordinated control policies. Different with the centralized approach, the distributed approach does not require a central station for control. The distributed approach is believed more promising and shows many advantages in achieving cooperative group performances, especially with high robustness, strong adaptivity, less system requirements. Pioneering works are generally referred to [1–2], thereafter consensus algorithms under various information-flow constraints have been studied in [3–17]. Recently, many researchers focus on the heterogeneous multi-agent systems. Adaptive control methods have been developed by adopting a fuzzy disturbance observer for heterogeneous multi-agent system which is consisted of heterogeneous nonlinear dynamic nodes, see [18] and reference therein. For excellent survey, please see [19].

* Corresponding author. Tel.: +86 24 83687762; fax: +86 24 83679605.
E-mail addresses: sunnyfengtao@163.com (T. Feng), hgzhang@ieee.org, zhg516516@gmail.com (H. Zhang), neuluo@neu.edu.cn (Y. Luo), lianghongjing99@163.com (H. Liang).

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Recently, a considerable amount of the existing literature is dedicated to finding optimal strategy subject to some given constraints [20–28]. Optimal consensus rate is one of the key optimization issues in distributed multi-agent coordination [19,29]. Consensus rate includes two indexes [30]: convergence speed which characterizes how fast consensus can be achieved and damping rate that evaluates the oscillating behaviors of the agents. Since the graph topology interplays with the system dynamics, the locations that the global closed-loop poles lie in are hard to determine even the closed-loop poles of each agent system are placed at specified region. For multi-agent systems consisting of single-integrator kinematics, the smallest nonzero eigenvalue of the Laplacian matrix determines the worst-case convergence speed [12], hence the convergence speed has been maximized by choosing optimal weights associated with edges [31,32]. Another common way focused on the case when all agents converge to the average of the initial states [33] which makes the optimization problems challenging if it is unknown. In [22,23], the inverse optimal design method was introduced to obtain the optimal distributed consensus protocols by constructing a global performance index. Inverse optimal problem was first raised by Kalman [36] and solved for the single-input linear systems, then was extended to the multi-inputs case in [38]. From the practical viewpoint, it is more practical in the LQ regulator design to give up the weight selection and design instead those state feedback controls which are optimal for some unknown weights, thereby simplifying the design procedures. In [37], the optimal regulator was designed by satisfying the pole assignment requirement which the LQR design method could hardly address. However, in [23], the authors only considered the consensus performance for a class of directed graphs. It is worth pointing out that optimal convergence for multi-agent systems consisting of general linear node dynamic on directed graphs is still an open problem [19].

One way to evaluate the performance of consensus protocols is to show how consensus depends on structural parameters of the communication graph by using the concept of consensus region [34], which is closely related to the gain margin of the feedback gain used in distributed protocols design. Obviously, a large consensus region is more desirable. In [35], an unbounded consensus region has been obtained by using the LQR optimal design method. However, the well developed LQR optimal consensus design methods are incapable of handling consensus rate issues. Improvements of the convergence rate and consensus region, both are vital characteristics in cooperative distributed design, should be analyzed together.

Motivated by the above facts, this paper aims to address issues of convergence rate and consensus region in cooperative distributed design by means of inverse optimal design methods [36–38]. The resulting distributed protocol is local optimal and its control gain contains two parameters, one is designed based on the asymptotic objective which is to place part of closed-loop poles to specified locations, and the other is tuned to ensure the asymptotic objective is achieved and meanwhile gradually eliminate the interplay from the graph topology, such that the consensus region extends to the whole right complex plane. Therefore, the multi-agent system can achieve specified consensus rate and desirable consensus region. What is more, the results clearly indicate that for the general linear continuous time multi-agent systems, the convergence speed has no upper bound, hence there is no optimal convergence rate on any given graph topology.

The main contributions of this paper are stated as follows:

- A novel and simple inverse optimal result is proposed which yields the optimal regulator that has an explicit parameterization and at least a specified guaranteed gain margin.
- The consensus problem for general linear multi-agent systems on fixed, directed graphs is solved by using the proposed inverse optimal design method. The resulting distributed protocol contains two parameters which can be designed such that the convergence speed and the damping rate are asymptotically achieved.
- The interplay from the graph topology can be gradually eliminated as the gain increases, thus the consensus region extends to the whole right complex z-plane. This is fairly significant if the number of agent nodes is large and the eigenvalues of the corresponding communication matrix is hard to determine or even troublesome to estimate.

The paper is organized as follows. Section 2 introduces the knowledge of graph theory. In Section 3, a new and simple time-domain solution of the inverse optimal problem of the LQ regulator is proposed. Such a time-domain solution leads to a fairly simple design procedure to the optimal partial pole placement. The LQ regulator designed places part of close-loop poles at specified locations exactly and has the specified gain margin. In Section 4, the cooperative distributed control problem, including leader following consensus problem and leaderless consensus problem, is addressed by the developed inverse optimal design method. Novel distributed cooperative design methods are developed. The resulting distributed consensus protocols are local optimal, and the asymptotic design objective is achieved with resort to possibly high gain regulators. The consensus rate and consensus region issues are both well addressed. A conclusion is given in Section 5.

Notations: Matrix $A > 0(\geq 0)$ means A is positive definite (semi-definite), $A < 0(\leq 0)$ means A is negative definite (semi-definite). The Kronnecker product is denoted by $\otimes$. The transposition of matrix A is denoted by A^T. The Hermitian transposition of matrix A is denoted by A^*. I_n denotes the n dimensional identity matrix in $\mathbb{R}^{n\times n}$. $1_n \in \mathbb{R}^n$ is the vector with all elements 1.

2. Preliminaries

2.1. Graph theory

Consider a weighted digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a nonempty finite set of N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ and the associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. An edge rooted at node j and ended at node i is denoted by (v_j, v_i), which means the information flows from node j to node i. The weight a_{ij} of edge (v_j, v_i) is positive, i.e., $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. In this paper, assume that there are no repeated edges and no self loops, i.e., $a_{ii} = 0, \forall i \in \mathcal{N}$, where $\mathcal{N} = \{1, 2, \ldots, N\}$. If