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Qualitative analysis of impulsive state feedback control to an algae-fish system with bistable property

Hongjian Guo [a,*], Lansun Chen [b], Xinyu Song [a,*]

[a] College of Mathematics and Information Science, Xinyang Normal University, Xinyang 464000, PR China
[b] Academy of Mathematics and Systems Science, Chinese Academy of Science, Beijing 100080, PR China

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ABSTRACT

A kind of consumer-resource system is proposed to describe the bidirectional interactions of the algae and the fish in an eutrophic water body. The dynamical properties of the proposed continuous system are given. For the bistable case, an impulsive semidynamical system with state feedback control, which depends on the biomass of the algae, is formulated and investigated to consider the feasibility of state feedback control for the aim of maintaining two species coexisting. The impulsive semidynamical system has three cases corresponding to three kinds of control measures: releasing fish, spraying algaecide, integrated control combining releasing fish and spraying algaecide. The existences of order-1 periodic solutions of three models are discussed by using successor function, respectively. The conditions under which the order-1 periodic solution is stable are given by using the Poincaré map and the analogue of Poincaré criterion. Mathematical results show that, for every one of three control strategies, there exists a range of control parameter in which the corresponding control is feasible. Finally, those mathematical results are verified by numerical simulations and the practical meanings are given.

1. Introduction

The algae can fix inorganic carbon through photosynthesis and make them into carbohydrates, which provides the basis of water productivity. As a producer in a fresh water ecological system, the algae is one of the food resource of the fish. But in an eutrophic water body, some kinds of algaes can grow quickly and release toxin into the water body, which have negative effects on the growth of the fish and other aquatic organisms. Therefore, the interactions of the fish and the algae are bidirectional. On the other hand, for the aims of protecting the fish and other aquatic organisms, it is necessary to control the biomass of the algae.

There are various of measures to decrease the biomass of the algae. Chemical measure is quick to remove the algae, but it can produce secondary pollution. It is generally believed that biological measure is safe. During the material transformation of food chain, every 1 kg fish needs to consumer about 100 kilograms of planktonic algae. Therefore, releasing some algophagous fish (e.g., Black carp, Grass carp) is one of the effective measures to control the excessive growth of algae in a fresh water. But if the fish is excessively released, plankton community will be damaged. For example, in Wuhan East Lake of China, the large amount of Grass carp were released to eliminate the water bloom and to increase the fishery production in 1970’s. The result is that the plankton communities were damaged [1]. In order to avoid this situation and to maintain two species coexisting, it is

* Corresponding authors. Tel.: +86 03766393516.
E-mail addresses: xbghj@163.com (H. Guo), xysong88@163.com (X. Song).

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necessary to carefully select the control measures and the relative control parameters (e.g., the amount of the released fish). In this paper, we will formulate a kind of impulsive semidynamical system and consider the feasibility and periodicity of impulsive state feedback control which depends on the biomass of the algae.

The bidirectional interactions of the algae and the fish are of consumer-resource(C-R) interactions. Holland and DeAngelis[2] developed a general theory for transitions between outcomes based on C-R interactions in which one or both species exploit the other as a resource. Simple models of C-R interactions indicated that the densities of the species alone could determine the fate of interactions. To test the influence of C-R interactions on the dynamics and stability of bi- and uni-directional C-R mutualisms, the simple models which link consumer functional response of one mutualistic species with the resources supplied by another is developed in paper [3]. Wang et al. [4] considered a predator-prey system of two species in which the predator consumes the prey and the prey has a harmful effect on the predator. By phase-portrait analysis and numerical simulations, it is demonstrated in paper [4] that interaction outcomes in the system may transition among predation, amensalism, competition, neutralism and commensalism. Varying initial densities of species population alone and varying one or more parameters (factors) can lead to the transition. Pal et al. [5] studied the effect of nutrient concentration and rate of toxin released by phytoplankton for the occurrence and termination of the planktonic bloom. Upadhyay et al. [6] investigated the dynamical complexities in two types of chaotic tri-trophic aquatic food-chain systems, where phytoplankton produce chemical substances known as toxins to reduce grazing pressure by zooplankton. There still are some references to investigate the C-R models, one can read the above papers and the references therein. Most of those models given in the above references considered the continuous models which have no the terms of impulsive state feedback control. We will consider an algae-fish system with impulsive state feedback control in which the algae as a food resource has negative effect on its consumer.

The ordinary differential equation with impulsive state effects is called as impulsive semidynamical system in Ref. [7] and semicontinous dynamical system in paper [8]. In those impulsive system, the conditions of impulsive effects depend on the state of the variables. The researchers have applied the impulsive semidynamical system on the biological mathematics fields, such as population system, turbidostat system and chemstat system. For examples, Tang and Cheke [9] proposed a state-dependent impulsive model for integrated pest management (IPM) and proved that there is no periodic solution with order larger than or equal to three, except for one special case, by using the properties of LambertW function and Poincaré map. Moreover, it is showed that the existence of an order two periodic solution implies the existence of an order one periodic solution [9].

Jiang and Lu [10] and Nie et al. [11] formulated and investigated the predator-prey models with impulsive state feedback control. The sufficient conditions for the existence and stability of semi-trivial solution and positive period-1 solution are obtained by using the Poincaré map and the analogue of the Poincaré criterion. Zeng et al. [12] generalized the Poincaré-Bendixson theorem of ordinary differential equation and gave an existence theorem of periodic solution of order one for a general planar autonomous impulsive system. Based on the ideas given in paper [12], some turbidostat systems and chemostat systems with impulsive state feedback control were proposed to investigate the periodicity of microorganism culture (e.g., [13–16]). Subsequently, Chen [8] gave the general ideas and methods such as the successor function to study the planar autonomous impulsive system. By using the successor function method, some mathematical models with impulsive state feedback control were formulated and investigated(e.g., [17–20]). The models in those papers have either the first integral or the stable equilibrium, or the limit cycle. But a few papers considered the impulsive semidynamical system with bistable property in which the positive equilibrium is a saddle point.

This paper will propose a kind of impulsive semidynamical system with bistable property to describe the evolution process of the algae and the fish under impulsive state feedback control, try to consider the feasibility and periodicity of impulsive state feedback control by investigating the existence and stability of periodic solution.

The rest of this paper is organized as follows. In Section 2, we will introduce a kind of simple consumer-resource system which can be viewed as a Kolmogorov-type system, and analyze its dynamical properties. For the bistable case, an impulsive semidynamical system is formulated. Some definitions and lemmas, the Poincaré map and the analogue of Poincaré criterion are also given in Section 2. The case of single releasing fish is discussed in Section 3 and the case of single chemical control is investigated in Section 4. The integrate control combining releasing fish and spraying algaecide is discussed in Section 5. Section 6 gives the numerical simulations and discussions.

2. Model formulation and preliminaries

2.1. Basic model

Since the fish consumes the algae and the algae has the negative effect on the fish, then we can use the consumer-resource(C-R) interaction model [2,3] to describe the bidirectional interactions of the algae and the fish. One of the bidirectional C-R interaction models can be written as the following form [21].

$$\begin{cases} \dfrac{dx_1}{dt} = x_1(r_1 + f_1(R_1(x_1, y_1)) - g_1(R_2(x_1, y_1)) - d_1x_1), \\ \dfrac{dy_1}{dt} = y_1(r_2 - g_2(R_1(x_1, y_1)) - d_2y_1), \end{cases} \tag{2.1}$$

where $x_1 = x_1(t)$ is assumed to be the consumer and $y_1 = y_1(t)$ the resource. The consumer x_1 feeds on the resource y_1. The resource y_1 has negative effect on the consumer x_1. The ratios r_1/d_1 and r_2/d_2 can be thought of as the carrying capacities in