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State space approach to boundary value problem for thermoelastic material with double porosity

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ABSTRACT

The present investigation is concerned with a boundary value problem in a homogeneous, isotropic, thermoelastic body with double porosity due to thermomechanical sources. After developing the mathematical formulation, a state space approach is applied to study the problem. As an application of the approach, normal force and thermal source have been taken to illustrate the utility of the approach. The expressions for the components of normal stress, equilibrated stresses and the temperature distribution are obtained in the frequency domain and computed numerically. Numerical simulation is prepared for these quantities and simulated results are depicted graphically for a particular model. A particular case of thermomechanical deformation in thermoelastic medium is deduced as a special case.

1. Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time have tried to overcome this difficulty and we see many theories of porous media in the literature. A brief historical background of these theories is given by de Boer [1,2].

Biot [3] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [4], de Boer and Ehlers [5] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity presented in a series of interesting papers given by Barenblatt et al. [6–9], where the double porosity model was

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proposed to express the fluid flow in hydrocarbon reservoirs and aquifers. In particular, Barenblatt [8] was concerned with the flow of slightly incompressible liquids through undeformable fissured rocks.

A fissured rock consists of a mass of porous blocks separated from each other by a system of fissures or fractures. Therefore, a fissured rock may be considered as a porous medium with double porosity: one porosity corresponds to the fissures and another to the pores of the blocks. The physical description of fissured rocks has been given by Gibson [10] and Pirson [11].

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super-saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Aifantis [12–16] introduced a multi-porous system and studied the mechanics of diffusion in solids. Wilson and Aifantis [17] presented the theory of consolidation with the double porosity. Khaled, Beskos and Aifantis [18] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Wilson and Aifantis [17]. Wilson and Aifantis [19] discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space. Beskos and Aifantis [20] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Moutsopoulos et al. [21] obtained the numerical simulation of transport phenomena by using the double porosity/ diffusivity continuum model.

Khalili et al. [22,23] studied the unified theory of flow and deformation in double porous media and presented a fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity structure. Pride and Berryman [24] studied the linear dynamics of double-porosity dual-permeability materials. Zhao and Chen [25], Svanadze [26] and Straughan [27] developed the basic equations for elastic materials with double porosity involving the displacement vector field, a pressure associated with the pores, and a pressure associated with the fissures. It is noted that in the equilibrium theory the fluid pressures become independent of the displacement vector field.

Svanadze [28-31] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity. Scarpetta and Svanadze [32,33] proved the uniqueness theorems in the theory of thermoelasticity for solids with double porosity and also obtained the fundamental solutions in the theory of thermoelasticity for solids with double porosity.

Nunziato and Cowin [34] developed a nonlinear theory of elastic material with voids. Later, Cowin and Nunziato [35] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They also considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Nunziato and Cowin have established a theory for the behavior of porous solids in which the skeletal or matrix materials are elastic and the interstices are voids of material.

Iesan and Quintanilla [51] used the Nunziato–Cowin theory of materials with voids to derive a theory of thermoelastic solids, which have a double porosity structure. This theory is not based on Darcy's law. In contrast with the classical theory of elastic materials with the double porosity, the double porosity structure in the case of equilibrium is influenced by the displacement field.

In recent years the state space description of linear systems has been used extensively in various areas of engineering, such as the analysis of control systems. The state space approach offers an attractive way to avoid the difficulties of the traditional linear model approach. The state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors. If the dynamical system is linear and time invariant, the differential and algebraic equations may be written in matrix form. The state-space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs.

Bahar and Hetnarski [36–41] investigated good number of problems in thermoelasticity by using state space approach. Also Anwar and Sherief [42], Ezzat et al. [43], El-Maghraby et al. [44,45], Youssef and Al-Lehaibi [46], Othman [47], Elisbai and Youseff [48], Youseff and Maatouk [49] and Sherief and El-Sayed [50] investigated different types of problems in different media by using state space approach.

We study the boundary value problem in thermoelastic material with double porosity due to thermomechanical sources. The state space approach has been used to solve it. The expressions for normal stress, equilibrated stresses and temperature change are obtained in closed form, computed numerically and represented graphically for normal force and thermal source. The comparisons are made in case of thermal with primary porous, thermal with double porous and thermoelastic theories. The result of the problem may be applied to a wide class of geophysical problem involving porosity and temperature change. The physical applications are encountered in the context of problem like ground explosion and soil industry, etc. The present investigation is also useful in the field of geomechanics where the interest is about the phenomenon occurring in the earthquake and measuring of displacement, stress, temperature distribution and equilibrated stresses due to the application of certain sources. The deformation at a point of the medium is significant to analyze the deformation field around the mining trammels and drilling crust