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Exact determinants and inverses of generalized Lucas skew circulant type matrices☆

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ABSTRACT

In this paper, we consider generalized Lucas skew circulant type matrices, including the skew circulant and skew left circulant. Firstly, we discuss the invertibility of generalized Lucas skew circulant matrix and present the determinant and the inverse matrix by constructing the transformation matrices. Furthermore, the invertibility of generalized Lucas skew left circulant matrix is also discussed. The determinant and the inverse matrix of generalized Lucas skew left circulant matrix are obtained respectively.

1. Introduction

Skew circulant matrices have important applications in various disciplines including image processing, communications, and signal processing. In [1], a new fast algorithm for optimal design of block digital filters (BDFs) is proposed based on skew circulant matrix. Compared with cyclic convolution algorithm, the skew cyclic convolution algorithm [2] is able to perform filtering procedure in approximate half of computational cost for real signals. In [3] two new normal-form realizations are presented by utilizing circulant and skew circulant matrices as their state transition matrices. The well-known second-order coupled form is a special case of the skew circulant form. Jiang et al. [4] displayed the different operators on linear vector space that are isomorphic to the algebra of $n \times n$ complex skew-circulant matrices. Li et al. [5] gave the style spectral decomposition of skew circulant matrix firstly, and then dealt with the optimal backward perturbation analysis for the linear system with skew circulant coefficient matrix.

Lately, some scholars gave the explicit determinant and inverse of the circulant and skew-circulant involving famous numbers. In [6], the explicit determinants of circulant and left circulant matrix involving Tribonacci numbers and generalized Lucas numbers are expressed in terms of Tribonacci numbers and generalized Lucas numbers only. Furthermore, four kinds of norms and bounds for the spread of these matrices are given respectively. Jiang et al. [7] discussed the invertibility of circulant type matrices with the sum and product of Fibonacci and Lucas numbers and presented the determinants and the inverses of these matrices. Jiang et al. [8] considered circulant type matrices with the k-Fibonacci and k-Lucas numbers and presented the explicit determinant and inverse matrix by constructing the transformation matrices. Jiang and Hong [9] gave exact form determinants of the RSFPLR circulant matrices and the RSLPFL circulant matrices involving Perrin, Padovan, Tribonacci, and the generalized Lucas number by the inverse factorization of polynomial. Bozkurt and Tam gave determinants and inverses of circulant matrices

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with Jacobsthal and Jacobsthal–Lucas numbers in [10]. Shen et al. considered circulant matrices with Fibonacci and Lucas numbers and presented their explicit determinants and inverses in [11]. Cambini presented an explicit form of the inverse of a particular circulant matrix in [12]. Jiang and Hong [13] discussed the invertibility of the Tribonacci skew circulant type matrices and present the determinants and the inverse matrices based on constructing the transformation matrices. Zhou and Jiang [14] gave the spectral norms of g-circulant matrices with classical Fibonacci and Lucas numbers entries. Jiang and Zhou [15] proposed a note for spectral norms of even-order r-circulant matrices.

The generalized Lucas sequences are defined by the following recurrence relations [16]:

$$\mathbb{L}_n = \mathbb{L}_{n-1} + \mathbb{L}_{n-2} + \mathbb{L}_{n-3},$$

where $\mathbb{L}_0 = 3, \mathbb{L}_1 = 1, \mathbb{L}_2 = 3, n \geq 3$.

Let t_1 , t_2 and t_3 be the roots of the characteristic equation $x^3 - x^2 - x - 1 = 0$, then we have

$$\begin{cases} t_1 + t_2 + t_3 = 1, \\ t_1t_2 + t_1t_3 + t_2t_3 = -1, \\ t_1t_2t_3 = 1. \end{cases} \tag{1}$$

then the Binet formulas of the sequences $\{\mathbb{L}_n\}$ have the form

$$\mathbb{L}_n = t_1^n + t_2^n + t_3^n. \tag{2}$$

In this paper, we consider skew circulant type matrices, including skew circulant and skew left circulant. We define a generalized Lucas skew circulant matrix which is an $n \times n$ complex matrix with following form:

$$\mathrm{Scirc}(\mathbb{L}_{1+m}, \mathbb{L}_{2+m}, \dots, \mathbb{L}_{n+m}) = \begin{bmatrix} \mathbb{L}_{1+m} & \mathbb{L}_{2+m} & \dots & \mathbb{L}_{n+m} \\ -\mathbb{L}_{n+m} & \mathbb{L}_{1+m} & \dots & \mathbb{L}_{n-1+m} \\ \vdots & \vdots & & \vdots \\ -\mathbb{L}_{2+m} & -\mathbb{L}_{3+m} & \dots & \mathbb{L}_{1+m} \end{bmatrix},$$

Besides, a generalized Lucas skew left circulant matrix is given by

$$\mathrm{Slcirc}(\mathbb{L}_{1+m}, \mathbb{L}_{2+m}, \dots, \mathbb{L}_{n+m}) = \begin{bmatrix} \mathbb{L}_{1+m} & \mathbb{L}_{2+m} & \dots & \mathbb{L}_{n+m} \\ \mathbb{L}_{2+m} & \mathbb{L}_{3+m} & \dots & -\mathbb{L}_{1+m} \\ \vdots & \vdots & & \vdots \\ \mathbb{L}_{n+m} & -\mathbb{L}_{1+m} & \dots & -\mathbb{L}_{n-1+m} \end{bmatrix},$$

where each row is a cyclic shift of the row above to the left.

2. Determinant and inverse of generalized Lucas skew circulant matrix

In this section, let $\mathbb{M}_{m,n} = \mathrm{Scirc}(\mathbb{L}_{1+m}, \mathbb{L}_{2+m}, \dots, \mathbb{L}_{n+m})$ be a generalized Lucas skew circulant matrix. Firstly, we give the determinant of the matrix $\mathbb{M}_{m,n}$. Afterwards, we discuss the invertibility of the matrix $\mathbb{M}_{m,n}$, and then we find the inverse of the matrix $\mathbb{M}_{m,n}$.

Theorem 1. *Let $\mathbb{M}_{m,n} = \mathrm{Scirc}(\mathbb{L}_{1+m}, \mathbb{L}_{2+m}, \dots, \mathbb{L}_{n+m})$ be a generalized Lucas skew circulant matrix, then we have*

$$\begin{aligned} \det \mathbb{M}_{m,n} = \mathbb{L}_{1+m} \cdot \Bigg[& [-(\mathbb{L}_{1+m} + s\mathbb{L}_{n+m}) - \sum_{i=1}^{n-2} \delta^i (\mathbb{L}_{n+1-i+m} + s\mathbb{L}_{n-i+m})]\kappa_1 \\ & + [(\mathbb{L}_{2+m} - \mathbb{L}_{1+m} + t\mathbb{L}_{n+m}) + \delta \cdot (\mathbb{L}_{1+m} - \mathbb{L}_{n+m} + t\mathbb{L}_{n-1+m}) \\ & + \sum_{i=1}^{n-3} \delta^{i+1} (\mathbb{L}_{n+1-i+m} - \mathbb{L}_{n-i+m} + t\mathbb{L}_{n-1-i+m})]\kappa_2 \Bigg], \end{aligned} \tag{3}$$

where $\mathbb{L}_n$ is the nth generalized Lucas number, and

$$\begin{aligned} \delta &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, s = -\frac{\mathbb{L}_{2+m}}{\mathbb{L}_{1+m}}, t = \frac{\mathbb{L}_{2+m} - \mathbb{L}_{3+m}}{\mathbb{L}_{1+m}}, \\ \kappa_1 &= (-\mathbb{L}_{1+m} + \mathbb{L}_{n+m} - t\mathbb{L}_{n-1+m})(\mathbb{L}_{1+m} + \mathbb{L}_{n+1+m})^{n-3} \\ &\quad + \sum_{i=2}^{n-2} (-1)^i (L_{1+m} + \mathbb{L}_{n+1+m})^{n-i-2} (\mathbb{L}_{n+2-i+m} - \mathbb{L}_{n+1-i+m} + t\mathbb{L}_{n-i+m}) \det A_{i-1}, \end{aligned}$$