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A family of ternary subdivision schemes for curves

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ABSTRACT

A family of ternary 3-point subdivision schemes is introduced and has a degree of smoothness C^1 continuity. A family of ternary 4-point subdivision schemes is derived and has degrees of smoothness C^1 and C^2 continuities. Also a ternary 4-point approximating subdivision scheme is proposed that generates the limiting curve of C^3 continuity. The eigenvalue analysis and generating function formalism are used to analyze the continuity properties of these subdivision schemes. Also these subdivision schemes are compared with the established subdivision schemes.

1. Introduction

Most work in the field of subdivision schemes has considered binary and ternary subdivision schemes. Subdivision schemes [2,3,8,13] provide useful methods for the creation of curves from the original data points.

In literature, a lot of papers have been published during the last decades. Dubuc [5] and independently Dyn et al. [8] proposed a binary 4-point interpolating subdivision scheme that generates the C^1 limiting curve. Deslauriers and Dubuc [3] analyze binary 2N-point subdivision schemes derived from polynomial interpolation. Weissman [16] introduced a binary 6-point interpolating subdivision scheme that generates the C^2 limiting curve. Hassan and Dodgson [9] derived ternary 3-point interpolating subdivision scheme that generates the C^1 limiting curve and ternary 3-point approximating subdivision scheme that generates the C^2 limiting curve. Siddiqi and Rehan [14] introduced a ternary 3-point approximating subdivision scheme that generates the C^2 limiting curve. Hassan et al. [10] developed a ternary 4-point interpolating subdivision scheme that generates C^2 limiting curves for the certain range of tension parameter. Siddiqi and Rehan [15] proposed a ternary 4-point interpolating subdivision scheme that generates the C^1 limiting curve. Ko et al. [12] presented a ternary 4-point approximating subdivision scheme that generates C^2 limiting curve. Beccari et al. [1] introduced a non-stationary ternary 4-point interpolating subdivision scheme that generates C^2 limiting curve with a single tension parameter.

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2. The subdivision schemes

Hassan and Dodgson [9] proposed a ternary 3-point interpolating subdivision scheme. The mask of the subdivision scheme [9] for the tension parameter $b = \frac{5}{18}$ is defined as

$$\alpha = \left[\ldots, \frac{-1}{18}, 0, \frac{5}{18}, \frac{7}{9}, 1, \frac{7}{9}, \frac{5}{18}, 0, \frac{-1}{18}, \ldots\right].$$

It has been proved that the ternary 3-point subdivision scheme generates the interpolating C^1 limiting curve.

In this paper, a ternary 3-point subdivision scheme is derived using the tension parameter in refinement rule p_{3i}^{k+1} as

$$\alpha = \left[\ldots, \frac{-1}{18}, \mu, \frac{5}{18}, \frac{7}{9}, (1-2\mu), \frac{7}{9}, \frac{5}{18}, \mu, \frac{-1}{18}, \ldots\right]. \tag{2.1}$$

The proposed ternary 3-point subdivision scheme generates an interpolating and approximating C^1 limiting curves for the different value of tension parameter μ.

The proposed ternary 3-point subdivision scheme generates an interpolating and approximating C^1 limiting curves for the different value of tension parameter μ.

Hassan et al. [10] developed a ternary 4-point interpolating subdivision scheme. The mask of the ternary 4-point subdivision scheme [10] for the tension parameter $\mu = \frac{1}{11}$ is

$$\alpha = \left[\ldots, 0, \frac{-4}{99}, \frac{-7}{99}, 0, \frac{34}{99}, \frac{76}{99}, 1, \frac{76}{99}, \frac{34}{99}, 0, \frac{-7}{99}, \frac{-4}{99}, 0, \ldots\right].$$

It has been proved that the ternary 4-point subdivision scheme generates the interpolating C^2 limiting curve.

In this paper, a ternary 4-point subdivision scheme is proposed using the tension parameter in refinement rule p_{3i}^{k+1} as

$$\alpha = \left[\ldots, -\mu, \frac{-4}{99}, \frac{-7}{99}, 4\mu, \frac{34}{99}, \frac{76}{99}, (1-6\mu), \frac{76}{99}, \frac{34}{99}, 4\mu, \frac{-7}{99}, \frac{-4}{99}, -\mu, \ldots\right]. \tag{2.2}$$

The mask of the refinement rule p_{3i}^{k+1}, using Lemma 2.2 of [11], is derived as

$$p_{3i}^{k+1} = \sum_{j=-2}^{2} L_j(0) p_j^k,$$

where

$$L_j(0) = \delta_{j,0} - \mu \prod_{k=-2, k\neq j}^{2} \frac{-2-k}{j-k}.$$

The proposed ternary 4-point subdivision scheme generates interpolating and approximating C^2 limiting curves for the different value of tension parameter μ.

Also a new ternary 4-point approximating subdivision scheme is introduced using cubic B-spline basis functions. Calculate the refinement rules p_{3i}^{k+1}, p_{3i+1}^{k+1} and p_{3i+2}^{k+1} at $\frac{1}{6}$, $\frac{1}{2}$ and $\frac{5}{6}$ respectively. The mask of the subdivision scheme is defined as

$$\alpha = \left[\ldots, \frac{1}{1296}, \frac{1}{48}, \frac{125}{1296}, \frac{113}{432}, \frac{23}{48}, \frac{277}{432}, \frac{277}{432}, \frac{23}{48}, \frac{113}{432}, \frac{125}{1296}, \frac{1}{48}, \frac{1}{1296}, \ldots\right]. \tag{2.3}$$

3. Convergence analysis—necessary conditions

Matrix formalism allows us to derive necessary conditions for a subdivision scheme to be C^m based on the eigenvalues of the subdivision matrices. Suppose the eigenvalues are λ_i, where $\lambda_0 = 1$ and $|\lambda_i| \geq |\lambda_{i+1}|$, for all $i \in N$. Then following Doo and Sabin [4] terminology, we can check the smoothness of the proposed subdivision schemes.

Consider the original vertices $\{A, B, C, D, E, F\}$ and the new vertices $\{a, b, c, d, e, f\}$, the subdivision matrix form of the ternary 3-point subdivision scheme (2.1) along the vertex is