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A combined binary 6-point subdivision scheme

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ABSTRACT

Combined binary 6-point interpolating and approximating subdivision scheme with tension parameters is analyzed. It is shown that the resulting curves are C^1, C^2 interpolating continuous and C^1, C^2, C^3 approximating continuous for different values of tension parameters. The role of the tension parameters in subdivision scheme are illustrated using a few examples.

1. Introduction

Subdivision schemes are widely used in many fields like computer graphics, computer aided geometric design and image processing etc. Most of the existing subdivision schemes are binary, ternary, stationary, symmetric and linear in form. The classic and the most popular interpolating subdivision schemes are binary 4-point interpolating subdivision scheme introduced by Dubuc [1] and independently Dyn et al. [3] with tension parameter. The limiting curves are C^1 continuous using these subdivision schemes. Tang et al. [9] verified the smoothness of binary 4-point interpolating subdivision scheme [1] using Laurent polynomial method. Tan et al. [12] presented a new 4-point subdivision scheme which keeps the second order divided difference at the old vertices unchanged when the new vertices are inserted. And the scheme generates the limiting curve of C^3 continuous.

In order to improve the smoothness of the subdivision schemes, Hassan et al. [4] introduced a ternary 4-point interpolating subdivision scheme that generates C^2 limiting curves. Siddiqi and Rehan [10] also developed a ternary 4-point interpolating subdivision scheme that generates the limiting curve of C^2 continuity. Pan et al. [13] introduced a new combined approximating and interpolating subdivision scheme which generates the limiting curve of C^2 continuous. Hormann and Sabin [6] constructed the subdivision schemes according to the properties such as support, precision set and so on. Dyn et al. [2] proposed a binary 4-point approximating subdivision scheme that generates the limiting curve of C^2 continuous. Augsdörfer et al. [7] took a geometric approach to generate subdivision curve for achieving different requirements. Siddiqi and Rehan [11] developed a new ternary 2N-point Lagrange subdivision scheme that generates the limiting curves of C^3, C^4 and C^5 continuities for $N = 2, 3$ and 4. Weissman [5] introduced a binary 6-point interpolating subdivision scheme that generates the limiting curve of C^2 continuity. Siddiqi and Ahmad [8] investigated the smoothness of the binary 6-point interpolating subdivision scheme [5] using Laurent polynomial method.

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In this paper, a step of subdivision can be taken as a highly simple step. By developing this step of the subdivision, we can generates families of limiting curves. Such improvement can lead to improved behavior of the subdivision scheme. For this, a new combined binary 6-point subdivision scheme is introduced using tension parameters in the mask f_{2i}^{k+1} and f_{2i+1}^{k+1}. These tension parameters give flexibility to generate families of approximating and interpolating limiting curves. A new combined 6-point approximating and interpolating subdivision scheme is defined as

$$\begin{aligned} f_{2i}^{k+1} &= a_0 f_{i-2}^k + a_1 f_{i-1}^k + a_2 f_i^k + a_3 f_{i+1}^k + a_4 f_{i+2}^k, \\ f_{2i+1}^{k+1} &= b_0 f_{i-2}^k + b_1 f_{i-1}^k + b_2 f_i^k + b_3 f_{i+1}^k + b_4 f_{i+2}^k + b_5 f_{i+3}^k. \end{aligned} \tag{1.1}$$

where

$$a_0 = a_4 = \frac{-1}{32}\alpha,\ a_1 = a_3 = \frac{1}{8}\alpha,\ a_2 = \left(1 - \frac{3}{16}\alpha\right),$$

$$b_0 = b_5 = ((1-\alpha)\beta),\ b_1 = b_4 = \left(\frac{-1}{16} - 3\beta(1-\alpha)\right),\ b_2 = b_3 = \left(\frac{9}{16} + 2\beta(1-\alpha)\right).$$

where α and β are tension parameters. The main purpose behind this idea using tension parameters in mask f_{2i}^{k+1} and f_{2i+1}^{k+1} basically is that it gives flexibility to generate families of approximating and interpolating curves. For $\alpha = 0$, the subdivision scheme (1.1) is the 6-point scheme with the tension parameter of Weissman [5]. For $\alpha = 0$ and $\beta = 0$, then the subdivision scheme (1.1) rebuilds the scheme in Dubuc [1]. For $\alpha = 0$ and $\beta = \frac{3}{4}$, the subdivision scheme (1.1) coincides with the scheme introduced by Hormann and Sabin [6]. For $\beta = 0$, the subdivision scheme (1.1) presented the scheme developed by Augsdörfer et al. [7].

2. Basic notion

A subdivision scheme with the initial values $f^0 = f_j^0 \in R : j \in Z$ defines recursively new discrete values $f^k = f_j^k \in R : j \in Z$ on finer levels by linear sums of existing values as follows

$$f_i^{k+1} = \sum_{j\in Z} a_{i-2j} f_j^k,\ k \in Z_+,$$

where the sequence $a = \{a_j : j \in Z\}$ is termed the mask of the given subdivision scheme. Here S denote the rule at each step and have the relation

$$f^k = S^k f^0. \tag{2.1}$$

A subdivision scheme S is said to be C^m if for the initial data $f^0 = \delta_{j,0} : j \in Z$, there exist a limit function $f = S^\infty f^0 \in C^m(R)$, $f \neq 0$, satisfying

$$\lim_{k\to\infty} \sup_{j\in Z} |f_j^k - f(2^{-k}j)| = 0.$$

For the convergent subdivision scheme S, the corresponding mask $\{a_j\}$,$j \in Z$ necessarily satisfies

$$\sum_{j\in Z} a_{2j} = \sum_{j\in Z} a_{2j+1} = 1.$$

Introducing a symbol called the Laurent polynomial $a(z) = \sum_{j\in Z} a_j z^j$ of a mask $\{a_j\}$,$j \in Z$ with finite support. The corresponding symbols play an efficient role to analyze the convergence and smoothness of subdivision scheme. Define the Laurent polynomial $a^{[k]}(z)$, $k \in N$, by

$$a^{[k]}(z) = a(z)a(z^2)\dots a(z^{2^{k-1}}) = \sum_{j\in Z} a_j^{[k]} z^j. \tag{2.2}$$

Using the coefficients $a_j^{[k]}$ in Eq. (2.2), the norm of the iterative subdivision scheme S^k in Eq. (2.1) is defined as

$$\|S^k\|_\infty = max\left\{\sum_{j\in Z} \left|a_{\gamma+2^k j}^{[k]}\right| : \gamma = 0, 1, \dots, 2^k - 1\right\}. \tag{2.3}$$

3. Smoothness analysis

Theorem 1. *A combined binary 6-point subdivision scheme defined in Eq. (1.1) converges and have smoothness, C^2 when $0 \le \alpha \le 1$ and $0 < \beta \le \frac{1}{50}$, C^3 when $0.6340 < \alpha \le 0.7531$ and $\beta = 0$.*

Proof. The generating function corresponding to the proposed combined binary 6-point subdivision scheme defined in Eq. (1.1) has the following sequence of coefficients

$$a = (a_i) = (\dots, b_0, a_0, b_1, a_1, b_2, a_2, b_3, a_3, b_4, a_4, b_5, \dots).$$