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A modified Perry's conjugate gradient method-based derivative-free method for solving large-scale nonlinear monotone equations☆

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ABSTRACT

In this paper, we propose a derivative-free method for solving large-scale nonlinear monotone equations. It combines the modified Perry's conjugate gradient method (I.E. Livieris, P. Pintelas, Globally convergent modified Perrys conjugate gradient method, Appl. Math. Comput., 218 (2012) 9197–9207) for unconstrained optimization problems and the hyperplane projection method (M.V. Solodov, B.F. Svaiter, A globally convergent inexact Newton method for systems of monotone equations, in: M. Fukushima, L. Qi (Eds.), Reformulation: Nonsmooth, Piecewise Smooth, Semismooth and Smoothing Methods, Kluwer Academic Publishers, 1998, pp. 355–369). We prove that the proposed method converges globally if the equations are monotone and Lipschitz continuous without differentiability requirement on the equations, which makes it possible to solve some nonsmooth equations. Another good property of the proposed method is that it is suitable to solve large-scale nonlinear monotone equations due to its lower storage requirement. Preliminary numerical results show that the proposed method is promising.

1. Introduction

In this paper, we consider solving the following nonlinear equations:

$$F(x) = 0, \tag{1.1}$$

where $F : R^n \longrightarrow R^n$ is continuous and monotone. By monotonicity, we mean

$$\langle F(x) - F(y), x - y \rangle \geq 0, \quad \forall x, y \in R^n. \tag{1.2}$$

Nonlinear monotone equations arise in various applications such as subproblems in the generalized proximal algorithms with Bregman distances [1]. Some monotone variational inequality problems can also be converted into systems of nonlinear monotone equations by means of fixed point maps or normal maps if the underlying function satisfies some coercive conditions [2].

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Among the iterative methods for solving problem (1.1), the Newton method, quasi-Newton methods, and their variants are attractive because of their fast local convergence property (see [3–6]). The main drawback of these methods, for large values of n, is that they need to solve a linear system of equations at each iteration using the Jacobian matrix or an approximation of it.

In recent years, great efforts have been made to find a solution of problem (1.1) especially for large-scale nonlinear monotone equations. Zhang and Zhou [9] combined the spectral gradient method [7] with the projection method [8] to solve nonlinear monotone equations. The method in [9] is globally convergent if the nonlinear equations are Lipschitz continuous. In addition, Yu et al. [11] proposed a spectral gradient procedure and a projection method, respectively, for nonlinear monotone equations with convex constraints.

Recently, researchers also aim at developing conjugate gradient based method for solving large-scale nonlinear equations. Cheng [21] first introduced a PRP-type method for nonlinear monotone equations, which is a combination of the well-known PRP conjugate gradient method and the hyperplane projection method [8]. Then some derivative-free methods were presented which also belong to the conjugate gradient scheme. Li and Li [16] proposed a three-term PRP based [13] derivative-free iterative method for solving the nonlinear monotone equations (1.1). Combined the three-term HS [14] methods with the projection scheme by Solodov and Svaiter [8], Yan et al. [17] proposed a derivative-free iterative method for solving the nonlinear monotone equations (1.1). In addition, Li [16] and Yan et al. [17] discussed the two-term version derivative-free method in Cheng [20] for solving the nonlinear monotone equations (1.1). Xiao and Zhu [15] presented a modified version of the CG_DESCENT method of Hager and Zhang [12] to solve the constrained nonlinear monotone equations. And under some mild conditions, they proved that their proposed method is globally convergent. More recently, Li [24] extended the modified Liu–Storey method [23] to solve the nonlinear monotone equations. There has been some research on applications (e.g. [25–28]).

The conjugate gradient-based derivative-free methods for nonlinear monotone equations possess some nice properties. First, due to the lower storage requirement, they are suitable for solving large-scale equations. Second, the methods are function value-based methods and can be applied to solve nonsmooth equations. Third, global convergence is achieved without the differentiability assumption on the equations. In addition, the methods are globally convergent even when the solution set is not a singleton.

In this paper, we will develop an efficient derivative-free algorithm for solving nonlinear monotone equations (1.1). Our work can be considered as a further research of the modified Perry's conjugate gradient method [18] in unconstrained optimization. Here, we extend it to solve nonlinear monotone equations with some little modifications. The rest of this paper is organized as follows. In Section 2, we first propose the method. In Section 3, we prove the global convergence of the proposed method. In Section 4, we report some numerical results to test the proposed method.

2. Motivation and the algorithm

In this section, we simply introduce modified Perry's conjugate gradient method in [18] for solving large-scale unconstrained optimization problems and describe the hyperplane projection method of Solodov and Svaiter. Combined with the hyperplane projection method, we extend modified Perry's conjugate gradient method [18] to solve large-scale constrained nonlinear equations (1.1).

In [18], Livieris and Pintelas proposed a modified Perry's conjugate gradient method. In order to guarantee that the proposed method generates descent directions, Livieris and Pintelas [18] exploit the idea of the spectral-type modified Fletcher–Reeves method [9,10] to the well-known Perry's conjugate gradient method [19]. Under suitable conditions, they establish the global convergence of our proposed method provided that the line search satisfies the Wolfe conditions. Numerical results demonstrate that the proposed method is promising.

Motivated by the modified Perry's conjugate gradient method [18] as well as by the projection scheme in Solodov and Svaiter [8], we propose an efficient derivative-free algorithm for solving nonlinear monotone equations (1.1). Firstly, we define the search direction as follows:

$$d_k = \begin{cases} -F_0, & \text{if } k = 0, \\ -\left(I + \beta_k^{\mathrm{MP}} \frac{F_k^T d_{k-1}}{\|F_k\|^2}\right) F_k + \beta_k^{\mathrm{MP}} d_{k-1}, & \text{if } k \geq 1, \end{cases} \tag{2.1}$$

where

$$\beta_k^{\mathrm{MP}} = \frac{F_k^T (y_{k-1} - \bar{s}_{k-1})}{d_{k-1}^T w_{k-1}} \tag{2.2}$$

$$w_{k-1} = y_{k-1} + \gamma \bar{s}_{k-1}, \gamma > 0, \quad y_{k-1} = F(z_{k-1}) - F(x_{k-1}), \quad \bar{s}_{k-1} = z_{k-1} - x_{k-1} = \alpha_{k-1} d_{k-1}. \tag{2.3}$$

For convenience, we call the method (2.1) and (2.2) as MPD method.

It is easy to see that the condition

$$F_k^T d_k = -\|F_k\|^2, \tag{2.4}$$

holds, for any line search.

About the hyperplane projection method in [8], note that by the monotonicity of F, for any $\tilde{x}$ such that $F(\tilde{x}) = 0$, we have

$$\langle F(z_k), \tilde{x} - z_k \rangle \leq 0$$