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Real iterative algorithms for a common solution to the complex conjugate matrix equation system☆

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ABSTRACT

In this paper, we propose a new algorithm for the computation of the common solution to the complex conjugate matrix equation system $A_iXB_i + C_i\overline{X}D_i = E_i,\ i = 1, 2, \ldots, N$. The algorithms only need finite iteration steps, requiring only real computation. Furthermore, the algorithm can be extended to solve a more general complex matrix equation system. Two numerical examples are given to illustrate the effectiveness of the proposed algorithm.

1. Introduction

In this paper, we consider the complex matrix equation system

$$A_iXB_i + C_i\overline{X}D_i = E_i, i = 1, 2, \ldots, N, \tag{1}$$

where $A_i, C_i \in \mathbb{C}^{m_i\times r}$, $B_i, D_i \in \mathbb{C}^{s\times n_i}$ and $E_i \in \mathbb{C}^{m_i\times n_i}$ are known matrices, $X \in \mathbb{C}^{r\times s}$ is the matrix to be determined and $\overline{X}$ denotes the matrix obtained by taking the complex conjugate of each element of X.

When $N = 1$, the matrix equation system (1) becomes the extended Sylvester-conjugate matrix equation $AXB + C\overline{X}D = E$, it was investigated in [13], and some explicit solutions were given. Obviously, the extended Sylvester-conjugate matrix equation includes the matrix equations $AX - \overline{X}B = C$ and $X - A\overline{X}B = C$ as its special cases. It was shown in [1] that the consistence of the matrix equation $AX - \overline{X}B = C$, was related to the consimilarity [3] of two matrices. Recently, in [11] some explicit expressions of the solution to the matrix equation $AX - \overline{X}B = C$ were established by the real representation of the complex matrix, and it was shown that there exists a unique solution if and only if $A\overline{A}$ and $B\overline{B}$ have no common eigenvalues. In addition, the matrix equation $X - A\overline{X}B = C$ was investigated in [4] with the aid of a real representation of complex matrix, the consistence and solutions of this equation were established in terms of real representation matrix equation. When the coefficient matrices and the unknown of the matrix equation $AXB + C\overline{X}D = E$ are all restricted to be real it becomes the matrix equation $AXB + CXD = F$ which plays a very important role in the stability analysis and controller design of descriptor linear systems [10,14]. When $N = 2$, $C_i = 0$ and $D_i = 0$, $i = 1, 2$, the matrix equation system (1) become the widely investigated case $(AXB, GXH) = (C, D)$. In [2,5,6], necessary and sufficient conditions for its solvability and expression of the solution were derived by means of generalized inverse. Recently, the finite iterative algorithm was presented to solve some special complex matrix equations in [7–9]. This algorithm can be implemented by initial coefficient matrices, and can provide a solution with finite iteration steps for any initial values.

In [12], an iterative algorithm is established for giving a common solution to the matrix equation system (1). However, the algorithm presented in [12] involves the complex computation, each step need to compute the conjugate matrix. In this paper,

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we propose a new algorithm to the matrix equation system (1), the algorithm only involves real computation, so that, we can avoid complex computation in every iteration. Furthermore, the new algorithm is also extended to solve a more general case.

Throughout this paper, the symbols $tr(A)$, A^H, A^T, $\overline{A}$ and $Re(A)$ are used to denote the trace, the conjugate transpose, the transpose, the conjugate and the real part of A respectively. The Frobenius norm of A is denoted by $\|A\|$, that is, $\|A\| = \sqrt{tr(A^TA)}$. This paper is organized as follows. In Section 2, some preliminaries are presented. In Section 3, we give the real iterative algorithms of the complex matrix equation system (1). In Section 4, the matrix equation system (1) is extended to a more general case. In Section 5, we present two numerical examples, which show that our new algorithms are effective.

2. Preliminaries

2.1. Real representation of a complex matrix

Given $A \in \mathbb{C}^{m\times n}$, it can be uniquely written as $A = A_1 + A_2\mathrm{i}$, $A_1, A_2 \in \mathbb{R}^{m\times n}$, $\mathrm{i} = \sqrt{-1}$. The real representation of A, denoted by A_σ, is defined in [4] by

$$A_\sigma = \begin{bmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{bmatrix} \in \mathbb{R}^{2m\times 2n}.$$

Let

$$P_j = \begin{bmatrix} I_j & 0 \\ 0 & -I_j \end{bmatrix}, \quad Q_j = \begin{bmatrix} 0 & I_j \\ -I_j & 0 \end{bmatrix},$$

where I_j is the $j \times j$ identity matrix. The real representation of a complex matrix possesses the following properties, which can be found in [4].

Lemma 2.1 (*The properties of the real representation*)**.**

(1) *If* $A, B \in \mathbb{C}^{m\times n}, a \in \mathbb{R}$, *then* $(A+B)_\sigma = A_\sigma + B_\sigma$, $(aA)_\sigma = aA_\sigma$, $P_mA_\sigma P_n = (\overline{A})_\sigma$.
(2) *If* $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{n\times r}$, *then* $(AB)_\sigma = A_\sigma P_n B_\sigma = A_\sigma(\overline{B})P_r$.
(3) *If* $A \in \mathbb{C}^{m\times m}$, *then A is nonsingular if and only if* A_σ *is nonsingular.*
(4) *If* $A \in \mathbb{C}^{m\times n}$, *then* $Q_mA_\sigma Q_n = A_\sigma$.

2.2. Finite iterative algorithm for the complex matrix equation system (1)

For the matrix equation system (1), in [12] the finite iterative algorithm was proposed to solve the common solution. The algorithm can be described as follows.

By [12], we have the following result.

Theorem 2.1. *If the matrix equation system (1) has a common solution, then Algorithm 1 can give a solution within finite iteration steps for any initial matrices $X(0)$ in absence of round-off errors.*

Algorithm 1 (Finite iterative algorithm for (1)).

1. Given initial value $X(0)$, calculate

$$R_i(0) = E_i - A_iX(0)B_i - C_i\overline{X(0)}D_i \in \mathbb{C}^{m_i\times n_i}, i = 1, 2, \ldots, N,$$
$$P(0) = \sum_{i=1}^{N}\left(A_i^HR_i(0)B_i^H + \overline{C_i}^H\overline{R_i(0)}\,\overline{D_i}^H\right), k = 0.$$

2. If $R_i(k) = 0$, $i = 1, 2, \ldots, N$, then stop; else, $k := k+1$.
3. Calculate

$$X(k+1) = X(k) + \frac{\sum_{i=1}^N \|R_i(k)\|^2}{\|P(k)\|^2}P(k),$$
$$R_i(k+1) = E_i - A_iX(k+1)B_i - C_i\overline{X(k+1)}D_i,$$
$$P(k+1) = \sum_{i=1}^{N}\left(A_i^HR_i(k+1)B_i^H + \overline{C_i}^H\overline{R_i(k+1)}\,\overline{D_i}^H\right) + \frac{\sum_{i=1}^N \|R_i(k+1)\|^2}{\sum_{i=1}^N \|R_i(k)\|^2}P(k).$$

4. Go to step 2.
