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Steady-state stock and group size: An approach of dynamic voluntary provisions of public goods

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ABSTRACT

This note analyzes the effect from group sizes (i.e., the number of consumers) on steady-state stocks in the model of dynamic voluntary provisions of public goods. The model follows Itaya and Shimomura [4]. We focus on feedback Nash equilibrium strategies and find that (1) it is possible to clarify the condition such that the steady-state stock decreases as the group size is larger when consumers take linear feedback Nash equilibrium strategies and (2) when consumers take nonlinear ones, the set of possible steady-state stocks is enlarged as the group size is bigger.

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1. Introduction

Differential game approaches are often employed to analyze the effect from consumptions on stock externalities. Fershtman and Nitzen [1] constructs a model regarding provisions of public goods by the differential game approach. Wirl [10] analyzes nonlinear feedback equilibria by a graphic approach (such approach is proposed by Tsutsui and Mino [9]) and Fujiwara and Matsueda [2] compares the steady-state stocks resulting from the cooperative solution, the open-loop equilibria and feedback equilibria. As extensions, Itaya and Shimomura [4], Ihori and Itaya [3] and Yanase [11] incorporate a private consumption into dynamic voluntary provision of public goods. Some recent researches about public goods can be confirmed by [12–14].

Following the model proposed by Itaya and Shimomura [4], this note analyzes the effect from group sizes (i.e., the number of consumers) on steady-state stocks in dynamic voluntary provisions of public goods. We focus on feedback Nash equilibrium strategies and compare the steady-state stocks under different group sizes. The main findings are as follows: (1) by linear feedback Nash equilibrium strategies, it is possible to clarify the condition such that the steady-state stock decreases as the group size is larger and (2) when consumers take nonlinear ones, the set of possible steady-state stocks is enlarged as the group size is bigger. In particular, Finding (1) follows the argument given by Olson [5] that public goods are less likely provided as the group size becomes larger.

This note is organized as follows: In Section 2, we present the model. Section 3 gives preliminaries. Sections 4 and 5 analyze effects from group sizes on steady-state stocks in terms of linear and nonlinear feedback Nash equilibrium strategies. The conclusion is given in Section 6.

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2. Models

The consumers set is given as $N = \{1, 2, \ldots, n\}$ where $n \geq 2$ refers to group size. Note that the group size n is constant during the game. Following Eq. (25) in Itaya and Shimomura [4], the long-run utility for each $i \in N$ is given as follows:

$$\int_0^\infty e^{-\rho t}\left[\alpha + \beta_1 c_i(t) - \frac{\gamma_1}{2}(c_i(t))^2 + \beta_2 G(t) - \frac{\gamma_2}{2}(G(t))^2\right]dt, \quad \alpha, \beta_1,\ \beta_2, \gamma_1, \gamma_2 > 0 \tag{1}$$

subject to

$$c_i(t) + g_i(t) = Y,\ c_i(t) \geq 0 \text{ and } g_i(t) \geq 0 \text{ for each } t \in [0, \infty);$$
$$\dot{G} = \frac{dG(t)}{dt} = \sum_{j=1}^{n} g_j(t) - \delta G(t);$$

the initial stock is given as G_0;

$$\beta_1 > \gamma_1 Y.$$

where (i) ρ stands for a common discount factor and $\rho \in (0, 1)$, (ii) $c_i(t)$ and $g_i(t)$ refer to the amount of the private consumption and the voluntary contribution for provisions of public goods (or contribution for short) respectively, (iii) Y stands for a constant income over time for each consumer, (iv) $G(t)$ stands for the stock of public goods, (v) $\dot{G}$ stands for the transition law of the stock, (vi) $\delta \in [0, 1)$ represents the natural depreciation rate and $\sum_{j=1}^{n} g_j(t)$ refers to the aggregate contribution and (vii) $\beta_1 > \gamma_1 Y$ is the assumption given by Itaya and Shimomura [4].

3. Preliminaries

We derive an auxiliary equation with respect to $c(G)$ and G. Following Fujiwara and Matsueda [2], the auxiliary equation can be used to derive feedback Nash equilibrium strategies.

Hamilton–Jacobi–Bellman equation corresponding to (1) is as follows:

$$\rho V(G) = \max_{c_i}\left[\alpha + \beta_1 c_i - \frac{\gamma_1}{2}(c_i)^2 + \beta_2 G - \frac{\gamma_2}{2}G^2 + V'(G)\left(nY - c_i - \sum_{j\neq i} c_j(G) - \delta G\right)\right]. \tag{2}$$

where $c_j(G)$ denotes the private consumption function of consumer j and $V(G)$ refers to a value function. First order conditions are

$$V'(G) = \beta_1 - \gamma_1 c_i(G). \tag{3}$$

Following the envelope theorem, differentiating Eq. (2) with respect to G yields the following equation:

$$\rho V'(G) = \left[\beta_2 - \gamma_2 G + V''(G)\left(nY - c_i - \sum_{j\neq i} c_j(G) - \delta G\right) + V'(G)\left(-\sum_{j\neq i} c'_j(G) - \delta\right)\right]\Bigg|_{c_i = c_i(G)}.$$

Throughout this note, we let consumers take symmetrical strategies, that is, $c_j(G) = c(G)$ for each $j \in N$. Substituting (3) into the above equation, we obtain an auxiliary equation as follows:

$$c'(G) = \frac{\beta_2 - \gamma_2 G - (\delta + \rho)\beta_1 + \gamma_1(\delta + \rho)c(G)}{\beta_1(n-1) + \gamma_1(nY - \delta G) - \gamma_1(2n-1)c(G)}. \tag{4}$$

4. Linear feedback Nash equilibrium strategies

A linear feedback Nash equilibrium strategy is represented by $(c(G), g(G))$ where $c(G) = FG + E$ and $g(G) = Y - c(G)$. Substituting $c(G)$ into Eq. (4), we can have the following two equations:

$$-F\gamma_1\delta G - F^2\gamma_1(2n-1)G = -\gamma_2 G + \gamma_1(\delta + \rho)FG$$

and

$$F\beta_1(n-1) + F\gamma_1 nY - F\gamma_1(2n-1)E = \beta_2 - (\delta + \rho)\beta_1 + \gamma_1(\delta + \rho)E.$$

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