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Statistical convergence behavior of affine projection algorithms

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ABSTRACT

Class of algorithms referring to the affine projection algorithms (APA) applies updates to the weights in a direction that is orthogonal to the most recent input vectors. This speeds up the convergence of the algorithm over that of the normalized least mean square (NLMS) algorithm, especially for highly colored input processes. In this paper a new statistical analysis model is used to analyze the APA class of algorithms with unity step size. Four assumptions are made, which are based on the direction vector for the APA class. Under these assumptions, deterministic recursive equations for the weight error and for the mean-square error are derived. We also analyze the steady-state behavior of the APA class. The new model is applicable to input processes that are autoregressive as well as autoregressive-moving average, and therefore is useful under more general conditions than previous models for prediction of the mean square error of the APA class. Simulation results are provided to corroborate the analytical results.

1. Introduction

In noise and echo cancellation, equalization, and beamforming, adaptive filtering techniques are widely used. A very popular algorithm is the normalized least mean square (NLMS) algorithm [1], which is computationally very simple and easy to implement. Unfortunately, for highly colored input signals – with a covariance matrix that exhibits a large dynamic range of eigenvalues – this algorithm suffers from slow convergence. Over the past three decades, computationally efficient, rapidly converging adaptive filtering algorithms [2–7] have been proposed to ameliorate this problem, which use multiple input vectors to compute the iterated direction of the adaptive filter. For example, Ozeki and Umeda [2] discovered the affine projection algorithm (APA) from the geometric viewpoint of affine subspace projections. Kratzer and Morgan [3] developed the partial rank algorithm (PRA), which addresses numerical conditioning. Sankaran and Beex [4] proposed NLMS with orthogonal correction factors (NLMS-OCF) based on the idea that the best improvement in weights occurs if successive input vectors are orthogonal to each other. Morgan and Kratzer [5] pointed out that all these algorithms, which were independently developed as a result of various interpretations and from different perspectives, can be viewed as a generalization of the NLMS algorithm that updates on the basis of multiple input signal vectors. Zhi [6,7] presented an affine projection algorithm with direction error (AP-DE) to solve the nonconformity between the iterated direction of the adaptive filter and the direction caused by the iteration error. Fast APA versions have been proposed as well [8–10]. We will refer to the entire class of algorithms as affine projection algorithms. In this paper, we will use the formulation by Ozeki and Umeda [2] to analyze the convergence behavior of the APA class.

Some work has been done to analyze the convergence behavior of the APA class. Reference [11] presented a definition of the APA based on the direction vector, but the error signal is still driven by the input vector. In this paper, on the other hand,

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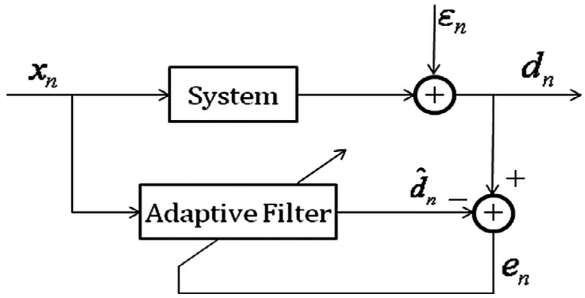

Fig. 1. Adaptive system identification scenario.

the error signal is driven by the direction vector. The direction vector behavior was investigated by means of carefully designed comparative experiments [12]. A quantitative analysis of the APA was presented in [13,14], which analyzes the mean weight error and the mean-square error (MSE) based on an independent and identically distributed input signal, which was proposed earlier [15] to analyze the convergence behavior of the NLMS algorithm. Subsequently, a unified treatment of the MSE, tracking, and transient performance was provided [16], based on energy conservation arguments and without restricting the regressors to specific models. A statistical analytical model for predicting the stochastic behavior of the APA class has been provided [17–19] for autoregressive (AR) inputs. A statistical analysis model was shown to analyze the AP-DE algorithm for AR input signals [6,7]. However, generally the AR input signal characteristics are not known. The above results motivate us to continue work on analyzing the APA class of algorithms for various types of input signals.

In this discussion follows the works in [13,14] and [17], but is not limited to AR process. The new model is applicable to input processes that are AR as well as autoregressive-moving average (ARMA). We analyze the quantitative statistical properties of the direction vector and propose some useful assumptions. We then study the convergence behavior of the weight error and a closed-form expression for the MSE of the APA class of algorithms, appropriate for ARMA input signal models, is obtained. The steady-state weight behavior is also determined. Finally, we show that our analytical results predict simulation results quite well.

Notations used in this paper are fairly standard. Scalars are denoted by plain lowercase or uppercase letters. Vector quantities are denoted by boldface lowercase letters and matrix quantities by boldface capital letters. Throughout the paper, the following notations are also adopted:

$(\bullet)^T$ Transpose of a vector or matrix;
$(\bullet)^H$ Hermitian transpose of a vector or matrix;
$(\bullet)^*$ Complex conjugate for scalars;
$E[\bullet]$ Expectation;
$tr(\bullet)$ Trace of a matrix;
$|\bullet|$ Absolute value of a quantity;
$real(\bullet)$ Real part of the complex number.

2. The APA class of algorithms

Fig. 1 show an adaptive filter used in the system identification mode. The wide sense stationary input process $\{x_n\}$, which is zero-mean, and the corresponding measured output $\{d_n\}$, possibly contaminated with the measurement noise $\{\varepsilon_n\}$, is measurable. The measurement noise $\{\varepsilon_n\}$ is zero mean white noise and is denoted by complex value. The input process is converted into input vectors $\{\mathbf{x}_n\}$, via a tapped delay line (TDL), and are defined as

$$\mathbf{x}_n = [x_n \quad x_{n-1} \quad \cdots \quad x_{n-N+1}]^T \tag{1}$$

The objective is to estimate an N-dimensional weight vector $\{\mathbf{w}_n\}$ using the most recent $(m+1)$ input vectors available at the nth instant; the latter define the input matrix $\{\mathbf{X}_n\}$ as follows

$$\mathbf{X}_n = [\mathbf{x}_n \quad \mathbf{x}_{n-1} \quad \cdots \quad \mathbf{x}_{n-m+1}] \tag{2}$$

The weight vector is adjusted so that the estimated output $\{\hat{d}_n\}$ is as close as possible to the measured output $\{d_n\}$ in the MSE sense. The APA is used to estimate these weights.

The adaptive filter implements the APA class of algorithms updates of the weight vector [2,11], as follows:

$$\hat{d}_n = \mathbf{w}_n^H \mathbf{x}_n \tag{3a}$$

$$e_n = d_n - \hat{d}_n \tag{3b}$$