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Matrix measure strategies for exponential synchronization and anti-synchronization of memristor-based neural networks with time-varying delays

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ABSTRACT

This paper is concerned with exponential synchronization and anti-synchronization of memristor-based neural networks. Under the framework of Filippov systems and a linear controller, the exponential synchronization and anti-synchronization criteria for memristor-based neural networks can be guaranteed by the matrix measure and Halanay inequality. The criteria are very simple to implement in practice. Finally, two numerical examples are given to demonstrate the correctness of the theoretical results. It is shown that the matrix measure can increase the exponential convergence rate and decrease the feedback gain effectively.

1. Introduction

Memristor, the fourth fundamental circuit element, was first proposed by Chua [1]. Since Hewlett-Packard research team realized the first practical memristor device [2,3], memristor-based neural networks have been paid more and more attention because they are suitable to emulate the human brain. The best property of memristor that should be emphasized is its hysteresis effects, in comparison to conventional resistor which cannot implement the function of memory. Such a property brings memristor-based neural networks a fruitful application in pattern recognition, signal processing, optimization and associative memories. It is reported that the number of equilibria in saturation regions of the neuronal state space of an n-neuron memristor-based cellular neural network significantly increases up to 2^{2n^2+n} compared with 2^n in a conventional cellular neural network without any memristor [4]. The application of memristor-based cellular neural networks for associative memories will undoubtedly improve the storage capacity.

Since the pioneering work of [5], synchronization of neural networks and complex networks has widely been investigated [6–12]. Up to now, there are some good works about dynamical analysis [13–18], synchronization [19–28] and anti-synchronization [29,30] of memristor-based neural networks . In [23], based on periodically intermittent control, they investigated the exponential synchronization of delayed memristive-based chaotic neural networks. The authors in [24] investigated the synchronization problem for memristor-based neural networks with the approaches of adaptive control and feedback control. In [29], a simple coupling control scheme is proposed for anti-synchronization of a class of memristive recurrent neural networks.

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It should be pointed out that the common method used in these papers is Lyapunov function (functional) approach. However, sometimes, it is difficult to construct a proper Lyapunov function for a complicated system or the criteria obtained are inconvenient to verify. What is more, most of the papers which investigated the exponential synchronization or anti-synchronization did not give the exact exponential convergence rate.

Motivated by the above discussions, this paper mainly focuses on the exponential synchronization and anti-synchronization of memristor-based neural networks with time-varying delays. Different from the past works, a new approach based on matrix measure and Halanay inequality is introduced to study the stability of the error systems. Sufficient criteria are derived under a linear feedback control scheme. Furthermore, the exponential convergence rate is given. Compared with Lyapunov function method in the proof, matrix measure [31–34] has the following advantages: (1) matrix measure can avoid constructing Lyapunov functions in the proof; (2) there is less restriction and requirement for the coupling matrix compared with matrix norm. The symmetry, negative (positive) definiteness and diagonal need of the coupling matrix are removed; (3) the results obtained by matrix measure method are usually less conservative than other approaches. The main contribution of this paper involves these three aspects. Furthermore, this is the first time to use matrix measure method to investigate the exponential synchronization and anti-synchronization of memristor-based neural networks with time-varying delays.

The remainder of this paper is organized as follows: In Section 2, model description and preliminaries are presented. In Section 3, exponential synchronization and anti-synchronization criteria are obtained by using matrix measure and Halanay inequality. In Section 4, we give two examples to show the usefulness of our results. Finally, the conclusions are drawn in Section 5.

2. Model formulation and preliminaries

In this paper, we consider the following memristor-based neural network with time-varying delays:

$$\dot{x}(t) = -C(x(t))x(t) + A(x(t))f(x(t)) + B(x(t-\tau(t)))f(x(t-\tau(t))) + J, \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ for $i = 1, 2, \ldots, n$ denotes the state variable associated with the neurons; n denotes the number of neurons; $C(x(t)) = \text{diag}\{c_1(x_1(t)), c_2(x_2(t)), \ldots, c_n(x_n(t))\}$, $i = 1, 2, \ldots, n$;

$$c_i(x_i(t)) = \begin{cases} c_i^*, & |x_i(t)| > T_j, \\ c_i^{**}, & |x_i(t)| < T_j, \end{cases}$$

where $c_i^* > 0$, $c_i^{**} > 0$, $c_i(\pm T_j) = c_i^*$ or c_i^{**}, $T_j > 0$ are the switching jumps; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$ and $f(x(t-\tau(t))) = (f_1(x_1(t-\tau_1(t))), f_2(x_2(t-\tau_2(t))), \ldots, f_n(x_n(t-\tau_n(t))))^T$ are the neuron activation functions of the neurons at time t and $t-\tau(t)$, $\tau_1(t), \ldots, \tau_n(t)$ are the time-varying delays and satisfy $0 \le \tau_i(t) \le \tau$, τ is a positive constant; $J = (J_1, \ldots, J_n) \in \mathbb{R}^n$ is a constant external input vector; $A(x(t)) = [a_{ij}(x_j(t))]_{n\times n}$, $B(x(t-\tau(t))) = [b_{ij}(x_j(t-\tau_j(t)))]_{n\times n}$, $i, j = 1, 2, \ldots, n$, are connection memristive weight matrix and the delayed connection memristive weight matrix, respectively:

$$a_{ij}(x_j(t)) = \begin{cases} a_{ij}^*, & |x_j(t)| > T_j, \\ a_{ij}^{**}, & |x_j(t)| < T_j, \end{cases} \qquad b_{ij}(x_j(t-\tau_j(t))) = \begin{cases} b_{ij}^*, & |x_j(t-\tau_j(t))| > T_j, \\ b_{ij}^{**}, & |x_j(t-\tau_j(t))| < T_j. \end{cases}$$

$a_{ij}(\pm T_j) = a_{ij}^*$ or a_{ij}^{**}, $b_{ij}(\pm T_j) = b_{ij}^*$ or b_{ij}^{**} for $i, j = 1, 2, \ldots, n$, a_{ij}^*, a_{ij}^{**}, b_{ij}^* and b_{ij}^{**} are all constants.

Throughout this paper, we consider (1) as the drive system and the corresponding response system is as follow:

$$\dot{y}(t) = -C(y(t))y(t) + A(y(t))f(y(t)) + B(y(t-\tau(t)))f(y(t-\tau(t))) + J + u(t), \tag{2}$$

where $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, $C(y(t)) = \text{diag}\{c_1(y_1(t)), c_2(y_2(t)), \ldots, c_n(y_n(t))\}$, $i = 1, 2, \ldots, n$;
$A(y(t)) = [a_{ij}(y_j(t))]_{n\times n}$, $B(y(t-\tau(t))) = [b_{ij}(y_j(t-\tau_j(t)))]_{n\times n}$,

$$c_i(y_i(t)) = \begin{cases} c_i^*, & |y_i(t)| > T_j, \\ c_i^{**}, & |y_i(t)| < T_j, \end{cases} \qquad a_{ij}(y_j(t)) = \begin{cases} a_{ij}^*, & |y_j(t)| > T_j, \\ a_{ij}^{**}, & |y_j(t)| < T_j, \end{cases}$$

$$b_{ij}(y_j(t-\tau_j(t))) = \begin{cases} b_{ij}^*, & |y_j(t-\tau_j(t))| > T_j, \\ b_{ij}^{**}, & |y_j(t-\tau_j(t))| < T_j, \end{cases}$$

and $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ is the controller.

The initial conditions of (1) and (2) are given by $x_i(s) = \phi_i(s)$, $y_i(s) = \psi_i(s)$, $s \in [t_0-\tau, t_0]$, $i = 1, 2, \ldots, n$, where $\phi(\cdot) = [\phi_1(\cdot), \phi_2(\cdot), \ldots, \phi_n(\cdot)]^T$, $\psi(\cdot) = [\psi_1(\cdot), \psi_2(\cdot), \ldots, \psi_n(\cdot)]^T \in C([t_0-\tau, t_0], \mathbb{R}^n)$. $\|\phi\|_p = \sup_{t_0-\tau\le s\le t_0} \|\phi(s)\|_p$ is used to denote the norm of a function $\phi \in C([t_0-\tau, t_0], \mathbb{R}^n)$. $\|\cdot\|_p$ is a vector norm and $p = 1, 2, \infty$. For $x \in \mathbb{R}^n$, the vector norm $\|\cdot\|_p$ is defined as $\|x\|_1 = \sum_{i=1}^n |x_i|$, $\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}$, $\|x\|_\infty = \max_{1\le i\le n} |x_i|$.

In this paper, since $a_{ij}(x_j(t))$ and $b_{ij}(x_j(t-\tau_j(t)))$ are discontinuous, solutions of all systems considered in this paper are handled in Filippov's sense [35]. Through the theories of differential inclusions and set-valued maps, from (1) and (2), it follows that

$$\dot{x}(t) \in -[\underline{C}, \overline{C}]x(t) + [\underline{A}, \overline{A}]f(x(t)) + [\underline{B}, \overline{B}]f(x(t-\tau(t))) + J, \tag{3}$$

and

$$\dot{y}(t) \in -[\underline{C}, \overline{C}]y(t) + [\underline{A}, \overline{A}]f(y(t)) + [\underline{B}, \overline{B}]f(y(t-\tau(t))) + J + u(t), \tag{4}$$