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Synchronization of fuzzy bidirectional associative memory neural networks with various time delays

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ABSTRACT

The problem of synchronization for fuzzy bidirectional associative memory (BAM) neural networks (NNs) with various time delays is formulated and investigated. The various delays consist of discrete delays, unbounded distributed delays and constant delay in the leakage term (i.e. 'leakage delay') are considered. Then, some sufficient conditions are presented to guarantee the global asymptotic stability of the error dynamical system by using Lyapunov–Krasovskii functional and linear matrix inequality (LMI) method. As a result, we achieved synchronization of master-slave (drive-response) fuzzy BAM NNs systems. Even, if there is no leakage delay, the obtained results are less restrictive to some known sufficient conditions. Moreover, the proposed results do not require the boundedness, differentiability and monotonicity of the activation functions, which can also be easily checked via the LMI solver in robust control toolbox in Matlab. Furthermore, using a parameter-dependent Lyapunov function approach the synchronization problem for polytopic uncertain fuzzy BAM NNs with various time delays is considered. The parameter uncertainties under consideration are assumed to belong to a fixed convex polytope. Finally, two numerical examples and simulations are given to illustrate the effectiveness of the derived theoretical results.

1. Introduction

It is well known that chaos synchronization is an important problem in nonlinear science. Synchronization means two or more systems which are either chaotic or periodic share a common dynamical behavior. It has been shown that this common behavior can be induced by coupling or by external forcing. As we all know, chaotic systems exhibit sensitive dependence on initial conditions. Because of this property, chaotic systems are difficult to be synchronized or controlled [5]. In recent years, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields [1–3], like applications in secure communications [4], biological networks and chemical reactions, since Pecora and Carroll [5] introduced a method to synchronization of two identical chaotic systems with different initial conditions. The idea of synchronization is making two chaotic systems (master–slave systems) oscillate in a synchronized manner. On the other hand, neural networks (NNs) have broad applications in many areas such as optimization, associative memories, machine learning and signal processing. As a result, there are increasing number of studies focusing on the dynamics of NNs, such as periodic solutions, stability analysis, bifurcations and chaos, and dissipativity [6–10]. In recent years, a scheme of secure communication based on the synchronization

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of unified chaotic system [4], Chuas circuits with multi-scroll attractors [11], hyperchaotic Chen and the unified chaotic systems [12] have been discussed. Especially for the application to secure communication, exponential synchronization of discontinuous chaotic systems via delayed impulsive control [13] and Chaotic lag synchronization of coupled time-delayed systems [14] have been investigated. Authors' in [15] dealt with the finite-time synchronization control for uncertain Markov jump neural networks with input constraints. Very recently, finite-time H_∞ synchronization for complex networks with semi-Markov jump topology [16], global synchronization of nonlinear coupled complex dynamical networks with information exchanges at discrete-time [17], and mean square exponential synchronization for impulsive coupled neural networks with time-varying delays and stochastic disturbances [18] have been investigated.

Cellular neural network (CNN), initially proposed by Chua and Yang in 1988 [19], has found important applications in various areas. CNN synchronization have been attracted much attention for the important applications in various areas such as in signal processing, especially in image processing, solving nonlinear algebraic and transcendental equations and some classes of optimization problems. Fuzzy cellular neural networks (FCNNs) first introduced by Yang and Yang [20,21] is another type cellular neural networks model, which combined fuzzy operation (fuzzy AND and fuzzy OR) with cellular neural networks. Recent studies have shown that the FCNN is very useful paradigm for image processing problems. The article [22] considered synchronization of delayed fuzzy cellular neural networks with all unknown parameters. Paper [23] dealt with the synchronization for a class of fuzzy cellular neural networks with delays and unknown parameters. Linear matrix inequality approach for synchronization control of fuzzy cellular neural networks with mixed time delays has been presented in [24].

A class of neural networks called bidirectional associative memory (BAM) NNs has been studied by some researchers [25,26] after it was originally introduced by Kosko [27,28]. Then, BAM NNs with delays have attracted considerable attention and have been widely investigated. It is composed of neurons arranged in two layers: the X-layer and the Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnection among neurons in the same layer. Through iterations of forward and backward propagation information flows between the two layers, it performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Therefore, it possesses good applications in the field of pattern recognition and artificial intelligence [29]. Likewise, other researchers have also investigated the problem of asymptotic stability of bidirectional associative memory neural networks with time-varying delays using delta operator approach [30].

Meanwhile, time delay will inevitably occur in the communication and response of neurons owing to the unavoidable finite switching speed of amplifiers in the electronic implementation of analog neural networks, so it is more in accordance with this fact to study the neural networks with time delays. Very recently, a leakage delay, which is the time delay in leakage term of the systems and a considerable factor affecting dynamics for the worse in the systems, is being put to use in the problem of stability for NNs [31]. However, so far, very little attention has been paid to NNs with time delay in the leakage (or "forgetting") term [32,33]. Such time delays in leakage terms are difficult to handle but have great impact on the dynamical behavior of NNs. Recently, the stability criteria for BAM neural networks has been studied by authors in [34] with leakage delays and probabilistic time-varying delays by using a model transformation and Lyapunov–Krasovskii functional. Also, the problem of global asymptotic stability of BAM fuzzy cellular neural networks with time delay in the leakage term, discrete and unbounded distributed delays has been discussed in [35]. The properties of M-matrix approach, the properties of fuzzy logic operator, Eigen space of the spectral radius of nonnegative matrices and delay differential inequality have been used by authors in [36] to derive the stability of fuzzy cellular neural networks with time delay in the leakage term and impulsive perturbations.

In recent times, more results on stability and synchronization of fuzzy cellular neural networks [20,22–24,33,37] and BAM NNs [26,38] with various time delays have been studied. Synchronization for delayed memristive BAM neural networks using impulsive control with random nonlinearities has been studied in [2]. However, to the best of authors' knowledge, the research on synchronization of fuzzy BAM NNs with discrete delays, unbounded distributed delays and leakage delay not yet investigated.

On the other hand, Lyapunov–Krasovskii functional approach is one of the most powerful tools to deal with the control problem of uncertain systems. This approach is practically necessary when dealing with nonlinear systems with time-varying parameters, the proposed techniques that is effective and insightful and the important classes of problems and special classes of functions the theory is supported by efficient numerical tools such as those based on linear matrix inequalities (LMIs). In stability analysis of uncertain systems, it is well known that the choice of an appropriate Lyapunov–Krasovskii functional is the key-point for deriving of stability criteria. In [15,39–42] authors show that the improvement can be made by choosing parameter-dependent Lyapunov functions. Motivated by the above discussion, in this paper, we will investigate the synchronization of fuzzy BAM NNs with various time delays. By constructing suitable Lyapunov–Krasovskii functional, which contains a triple integral term and free-weighting matrices method, and employing some analysis techniques, a sufficient condition is derived to ensuring the synchronization of fuzzy BAM NNs with various time delays. The main contributions of the paper as follows: (1) In this manuscript we have considered fuzzy BAM NNs with discrete delays, unbounded distributed delays and constant delay in the leakage term. (2) The sufficient conditions for the synchronization of fuzzy BAM NNs are derived in terms of LMIs, which can be solved by using Matlab LMI solver. (3) Moreover, this method is then extended to a system with polytopic-type uncertainties by using a parameter-dependent Lyapunov functional and LMI approach. The derived result is of great interest in the design and applications of neural circuits. Finally, two numerical examples and simulations are given to show the effectiveness of our result.

Notations: Throughout this paper, the superscript T denotes the transposition; B^{-1} denotes the inverse of B; $|B| = [|b_{ij}|]_{n\times m}$; $B > 0$ or $B < 0$ denotes that the matrix B is a symmetric and positive definite or negative definite matrix; I denotes the identity matrix of appropriate dimension; $\mathbb{R}^n$ and $\mathbb{R}^m$ denote the n-dimensional and m-dimensional Euclidean space respectively; $\mathcal{I} = 1, 2, \ldots, n,\ \mathcal{J} = 1, 2, \ldots, m$. Moreover, the notation $*$ always denotes the symmetric block in one symmetric matrix.