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Effect of heterogeneous sub-populations on the evolution of cooperation

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ABSTRACT

Understanding origin of cooperation denotes one of the most challenging conundrums across myriad disciplines. Different from previous assumption that all the individuals have identical dilemma, here we put forward a heterogeneous sub-population model on regular lattices and complex networks: players face different dilemmas and cooperation tendency inside and outside sub-population. In details, each agent will play different games with the opponents from his own sub-population or from a different sub-population. By means of numerous simulations, we find that sub-population is a useful way to resolve social dilemmas, which is universally effective for interaction topology. Interestingly, less sub-populations can guarantee the optimal environment of cooperation, the continuous increment of sub-population number in turn impedes the evolution of cooperation, which though seems better than the traditional scenario (namely, pure prisoner's dilemma or snowdrift game on network). Moreover, the fraction of cooperation also depends on the related scale of such sub-populations. From the viewpoint of microscopic dynamics, we further explore the transition probability of different strategies and the organization of cooperator clusters. Because this framework of heterogeneous sub-populations is close to realistic life, we hope that it can provide new insight to resolve the social dilemmas.

1. Introduction

From cellular organisms to human beings, cooperation behavior is ubiquitous and plays a key role in the evolution of species [1,2], which, however, seems inconsistent with the Darwin's prediction that any behavior bringing benefit to others is not beneficial for the evolution of itself [3]. Thus, how to explain the origin of cooperation becomes a basic question in evolutionary biology and other disciplines. Borrowing from the new emerging technology and analysis methods, evolutionary game theory has been proved to provide a useful framework to this issue. Among the existing attempts, the prisoner's dilemma game, as a simple and paradigmatic metaphor, has attracted great attention from theoretical and experimental viewpoints. In its basic version, two players decide to cooperate or defect simultaneously. Both gain the reward R (punishment P) for mutual cooperation (defection). If however, one defector meets a cooperator, the former gets the temptation T and the latter is left with the sucker's payoff S. The payoffs satisfy the ranking $T > R > P > S$ and $2R > T + S$, from which it is clear that defection is the best (i.e., Nash Equilibrium) irrespective of the fact that mutual cooperation can bring higher collective benefit [4–6]. On the other hand, the snowdrift game, as an alternative of the prisoner's game, takes places if there is a slight change of payoff ranking $T > R > S > P$, where individual's best strategy depends on his opponent's action. This leads to the increase of strategy pairs between cooperation and defection. In

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both social dilemmas, the mass emergence of cooperation is greatly difficult, which is unlike the case of harmony game: mutual cooperation bringing the highest benefit.

To overcome the unfortunate social dilemmas, a great number of scenarios have been identified to promote the evolution of cooperation. Typical examples include diluted environment [7], mobility [8–10], fitness evaluation [11], punishment and reward [12], reputation [13–15], asymmetric payoff [16], differences in evolutionary time scales [17], additional noise [18,19]. While recently, Nowak attributed all these scenarios to five mechanisms: kin selection, direct reciprocity, indirect reciprocity, group selection and network reciprocity [20]. While among them, network reciprocity (also called spatial reciprocity) has attracted the greatest attention, which demonstrates the effect of spatial topology structure on the promotion of cooperation [21–23]. Along this seminal finding, the role of spatial network is further extended, such as heterogeneous networks [24–27], hierarchical structure [28], adaptive networks [29] and multilayer networks [30–32]. We can look at some examples more specially, in Ref. [33], where two interdependent networks are correlated by the utility function (i.e., a re-definition of individual payoff), it is shown that strong coupling would promote cooperation and also brings the spontaneous symmetry breaking (namely, cooperation level is unequal in two networks). If different network layers supported two related yet different games, cooperation could also reach a completely dominant state [34] (see Refs. [35,36] for a recent review as well).

In spite of the fruitful achievements during the past decades, the above literatures simply assume that the environment among players is identical (namely, each pair of players faces the same dilemma). However, in real scenarios, a whole population can usually be divided into several different sub-populations [37,38], each of which has completely different dilemmas inside and outside itself. Inside such a sub-population, players, like friends and family members, are familiar with each other, and more likely to choose mutual cooperation. At variance, outside the sub-population, the inclination of choosing cooperation obviously declines, which resorts to the case of well-known prisoner's dilemma or snowdrift game. In particular, similar idea was also considered previously. For example, in Ref. [39], which carried out the study based on the vision approach, it was found that sub-populations actually play a critical role in the organization of cooperation society. Moreover, Weibull also summarized two heterogeneous sub-populations with conflicting interactions [40]. However, these aforementioned works ignore the difference of dilemmas (or individual inclination to cooperation) inside and outside sub-populations. An interesting question naturally poses itself, which we aim to address in what follows. Namely, if there exist different dilemmas for players inside and outside sub-populations, does it affect cooperation? If yes, how many sub-populations are the best for organization of cooperation?

Aiming to resolve the above questions, here we consider the evolutionary games with heterogeneous environment of sub-populations, inside which harmony game is involved and outside which prisoner's dilemma or snowdrift game is used to feature individual interaction. Via numerous computation simulations, it is clearly found that the consideration of sub-populations promotes cooperation to an extremely high level. It is interesting that the continuous increment of sub-population number conversely breaks down the evolution of cooperation. Moreover, it is also unveiled that the more obvious the difference between the ratio of each sub-population, the larger the fraction of cooperation. With regard to these observations, we provide the microscopic analysis based on the transition probability of strategies and cluster expansion.

2. Methods

Due to the existence of sub-populations, we consider different tendency of choosing cooperation inside and outside sub-populations. The former case usually indicates that players in the same sub-population have good relationships and are familiar with each other, thus they are more likely to hold mutual cooperation. The latter case implies that agents come from completely different sub-populations and their inclination of choosing cooperation is not as strong as the former case. In this case, prisoner's dilemma game and snowdrift game become suitable candidates for the latter case. Here the payoffs of both games follow the standard procedure to govern the social dilemmas. The prisoner's dilemma game is characterized by the temptation $T = b$, reward for mutual cooperation $R = 1$, and punishment P as well as the sucker's payoff S equaling to 0, whereby $1 < b < 2$ ensures a proper payoff ranking $T > R > P \geq S$ [49]. On the other hand, we also employ the snowdrift game with the payoffs $T = 1 + r, R = 1, S = 1 - r$ and $P = 0$, thus satisfying the ranking $T > R > S > P$, where $0 < r < 1$ represents the so-called cost-to-benefit ratio. In particular, parameters b and r also denote the dilemma strength in prisoner's dilemma and snowdrift games, respectively. With regard to tendency inside sub-populations, we consider the harmony game, where the mutual cooperation as final equilibrium. For decreasing the number of payoff parameters yet without changing payoff ranking $R > T > S > P$, we take the rescaled payoffs: $T = 1.0, S = 0.5, P = 0$ and $R = b$ or $1 + r$ when confronting prisoner's dilemma game or snowdrift game outside sub-populations.

As the interaction network, we use either regular square lattice, small-world network, or the random regular graph (RRG) with size of L^2 and periodic boundary conditions. Each node x is initially assigned as cooperator ($s_x = C$) or defector ($s_x = D$) with equal probability. Besides, due to the consideration of different dilemmas, each player is also randomly encapsulated into one sub-population α, whose related scale (or ratio) in the whole population is f_α $(0 \leq f_\alpha \leq 1)$ and $\sum_{\alpha=1}^{N} f_\alpha = 1$, where N is the total number of sub-populations. It is worth mentioning that this ratio will keep constant during the whole process, irrespective of individual strategy. This setup is similar to the study of heterogeneous ability not involving any co-evolution mechanism [50]. In particular, for $N = 0$, there is no sub-population and each agent plays the prisoner's dilemma or snowdrift game (no harmony game) with all his neighbors, namely, the so-called "traditional case" (pure prisoner's dilemma or snowdrift game) [49,51]. Although $N = 1$ makes all the players face the identical dilemma as well, the underlying interaction model becomes harmony game (according to the definition of sub-population), which is greatly different from the traditional case. Since cooperation is completely dominant in harmony game, we will pay the main attention to comparing the evolution of cooperation between the setup of more sub-populations (namely, $N \geq 2$) and traditional case. In fact, the larger the value of N, the more evident the