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A new optimal portfolio selection model with owner-occupied housing☆

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ABSTRACT

This paper develops a new dynamic optimal portfolio selection model with owner-occupied housing. Such a model has three features: (1) the objective of an agent is to minimize the deviation of her wealth to a certain pre-set financial target by selecting a suitable portfolio strategy; (2) the house price is modeled by a stochastic differential equation with Poisson jump; (3) both full information and partial information are considered. The optimal portfolio strategies with the associated optimal performance functionals are completely and explicitly obtained in terms of some methods arising from stochastic optimal control and backward stochastic differential equation. A numerical example is used to demonstrate the theoretical results.

1. Introduction

Stochastic optimal control theory plays an important role in many fields such as finance, economics and real estate. One of the central problems in the theory of economic growth is how the resources should be split among investment, consumption and stochastic expenditure/income. About forty years ago, Merton [10] worked out the well-known stochastic security market models and studied some utility optimization problems. Recently, Cocco [5] showed that housing can reduce the benefits of equity market participation. Chu [4] studied a portfolio choice and asset pricing problem with owner-occupied housing. Fan et al. [7] established a utility indifference model for evaluating some prices related to forward transactions in a certain housing market. Note that the above works merely focused on certain assumptions as follows: (1) discrete time models; (2) "usual" utility functionals (i.e., concave functions); (3) the dynamic programming principle was used to treat their problems.

In fact, the agent is also concerned with the derivation of her wealth from a certain pre-set financial target. Such a class of problems has important values in both theoretical and practical applications. But rather little research attention has been paid along this line. Keeping this in mind, this paper aims to address an optimal portfolio choice problem with owner-occupied housing. In detail, we assume that the agent continuously trades a "risk-free" asset and a "risky" asset, and does not buy a house until its construction is completed. The objective of the agent is to minimize the deviation of her wealth to a certain pre-set

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financial target by selecting an optimal portfolio strategy. Our aim is to find a closed-form solution of the optimal portfolio strategy in the full and partial information situations, respectively.

This paper is organized as follows. In Section 2, some basic market models are introduced. In Section 3, the full information optimal portfolio choice problem with owner- occupied housing is first formulated, followed by the closed-form optimal portfolio strategy derived via the method of "completion of squares" and the theory of backward stochastic differential equations (BSDEs, for short). In Section 4, some particular cases are given. In Section 5, the partial information counterpart of Section 3 is studied. Section 6 concludes this paper.

2. Basic model

We begin with a finite time horizon $[0, T]$ for $T > 0$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\le t\le T}, \mathbb{P})$ be a filtered complete probability space which carries a two-dimensional standard $\mathcal{F}_t$-Brownian motion $V = (V_t^0, V_t^1)_{0\le t\le T}$, and a one-dimensional centralized $\mathcal{F}_t$-Poisson process $\tilde{N}_t = N_t - \lambda t$, where N_t is a Poisson process with $\mathbb{E}N_t = \lambda t$ and $\lambda > 0$ is a constant. Throughout this paper, we denote by $\mathbb{R}^n$ the n-dimensional Euclidean space, by $\mathcal{L}^2_{\mathcal{F}}(0, T; S)$ the set of all S-valued, $\mathcal{F}_t$-adapted and square-integrable processes, and similarly with the other sigma algebras and Euclidean spaces.

Now consider an agent who can continuously trade three types of assets without transaction costs. The first type is a "risk-free" bond, whose price is governed by

$$dB_t = r_t B_t dt,$$

where r_t is the interest rate of the bond. The second type is a "risky" stock, whose price is log-normally distributed, i.e., satisfies the stochastic differential equation (SDE, for short)

$$\frac{dP_t^0}{P_t^0} = \mu_t^0 dt + \sigma_t^0 dV_t^0,$$

where μ_t^0 and σ_t^0 are the appreciation rate of return and volatility coefficient of the stock, respectively. For simplicity, we assume that the coefficients $\mu_t^0 \ge r_t > 0$, σ_t^0 and $\frac{1}{\sigma_t^0}$ are bounded and deterministic processes. The third type is owner-occupied housing, whose price is assumed to be modeled by the SDE with a Poisson process

$$\begin{cases} dP_t^1 = \mu(t, P_t^1)dt + \sigma(t, P_t^1)dV_t^1 + \delta(t, P_{t-}^1)d\tilde{N}_t, \\ P_0^1 = p_0^1 > 0. \end{cases} \tag{1}$$

Here $\mu(t, P_t^1)$, $\sigma(t, P_t^1)$ and $\delta(t, P_t^1)$ are the drift coefficient and the diffusion coefficient of the house price; dV_t^1 describes the "normal events" in the house price, and can be interpreted as a combination of the idiosyncratic risk specific to housing asset, the local risk related to metropolitan area, and so on (see e.g. Bayer et al.[1]); $d\tilde{N}_t$ captures the "rare and sudden events" in the price. Note that the model (1) is standard and extensively used in the context of mathematical finance. See e.g. Merton [10], Cont and Tankov [6] for more details. Also, the model (1) is general but it contains several special models which are usually used in the field of real estate [4,7]. For example, if $\mu(t, P_t^1) = \mu_t^1 \mu_0(P_t^1)$, $\sigma(t, P_t^1) = \sigma_t^1 \sigma_0(P_t^1)$ and $\delta(t, P_{t-}^1) \equiv 0$, then P_t^1 satisfies a geometric Brownian motion; if $\mu(t, P_t^1) = c(\theta - P_t^1)$, $\sigma(t, P_t^1) = \sqrt{P_t^1}$ and $\delta(t, P_{t-}^1) \equiv 0$, then P_t^1 satisfies a square-root process. In what follows, we assume that for any (t, x), (t, x_1), $(t, x_2) \in [0, T] \times \mathbb{R}$, there is a constant $k > 0$ such that

$$|\mu(t, x)| + |\sigma(t, x)| + |\delta(t, x)| \le k(1 + |x|),$$

and

$$|\mu(t, x_1) - \mu(t, x_2)| + |\sigma(t, x_1) - \sigma(t, x_2)| + |\delta(t, x_1) - \delta(t, x_2)| \le k|x_1 - x_2|.$$

From Ikeda and Watanabe [8], the model (1) admits a unique (strong) solution $P^1 \in \mathcal{L}^2_{\mathcal{F}}(0, T; \mathbb{R})$.

3. An optimal portfolio choice problem with full information

3.1. Problem formulation

We denote by X_t the wealth from investment in the financial market, by π_t the amount that the agent invests in the stock. Then, the agent has $X_t - \pi_t$ savings in the bank. Given the above notations, it is easy to see that X_t is modeled by

$$\begin{cases} dX_t = (X_t - \pi_t)\dfrac{dB_t}{B_t} + \pi_t \dfrac{dP_t^0}{P_t^0} \\ \qquad = [r_t X_t + (\mu_t^0 - r_t)\pi_t]dt + \sigma_t^0 \pi_t dV_t^0, \\ X_0 = x_0 > 0, \end{cases} \tag{2}$$

where x_0 is the initial endowment of the agent. Suppose that the agent does not purchase the house until its construction is completed at time t, and she pays cash for the house. Then her wealth in the financial market excluding the house is $X_t - P_t^1$. Furthermore, we confine ourselves to the case

$$x_0 - p_0^1 \ge -c_0,$$