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A class of differential quadratic programming problems[☆]

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ABSTRACT

A class of differential quadratic programming problems in finite dimensional spaces is introduced in this paper. First, the existence of solutions for the differential quadratic programming problem is established under some suitable assumptions. Second, an algorithm for solving the differential quadratic programming problem is given and the convergence analysis for the algorithm is shown. Finally, some numerical experiments are reported to verify the validity of the proposed algorithm.

1. Introduction

In electricity markets, many researchers pay attention to the problem of how to reduce the network loss of power (see, for example, [9,11,12,14]). One of the effective models to characterize the real power economic dispatch problem in an electrical network is a quadratic program, which is formulated as follows: Find $u \in R^n$ such that,

$$\begin{aligned} \text{Minimize} \quad & u^T A u + h^T u + e \\ \text{subject to} \quad & u \in K. \end{aligned} \tag{1.1}$$

We list the significance of some symbols in (1.1) as follows:

- u denotes the active power vector of generators, and the components of u are the power produced by each generator in the system. u^T is the transposition. The active power vector of generators describes the electrical energy used for power consumption, which is converted to heat, light, mechanical energy and so on.
- $A \in R^{n\times n}$, $h \in R^n$ and $e \in R$ denote the coefficients of consumption characteristics. The coefficient of consumption characteristics describes the relationship between the power consumption of generating equipment and the one of useful work. More specifically, A, h and e are parameters so that the objective function in (1.1) describes the fraction of the total power loss which is mainly due to resistance.
- n denotes the number of generators in a power distribution system. For example, there are 32 generators in Three Gorge Power Station in China. $K \subset R^n$ denotes the constraint set for the active power vector which consists of power balance constraints,

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line constraints, power limit constraints and so on. In fact, we mainly consider power limit constraints. For example, the power limit constraint of each generator in Three Gorge Power Station is 7×10^5 *KW*.

It is well known that resistance results in the network losing power and coefficients A, h and e are influenced by resistance. Moreover, resistance is mainly determined by the conductor material properties and temperature. Generally speaking, the higher the temperature, the greater the resistance. Since the conductor temperature will be increasing as the transmission process continues, the coefficients A, h and e are not constant. It is hence more reasonable to consider the following model with a continuous-time system rather than problem (1.1), i.e.,

$$\begin{cases} \text{Minimize} & u(t)^T A(r(t))u(t) + h(r(t))^T u(t) + e(r(t)) \\ \text{subject to} & u(t) \in K \\ \text{where} & \dot{r}(t) = f(r(t), u(t)), \quad r(0) = r^0, \end{cases} \tag{1.2}$$

where $r(t)$ denotes the resistance at time $t \in [0, T]$, $\dot{r} \equiv \frac{dr}{dt}$ is the time-derivative of the function $r(t)$, f: $R \times R^n \to R$, A: $R \to R^{n\times n}$, h: $R \to R^n$ and e: $R \to R$ are given mappings and r^0 denotes the initial data. Problem (1.2) is called a differential quadratic programming problem, abbreviated as DQP.

There are many results concerned with quadratic programming problems without the constraint of differential equations. For example, a branch and bound method for box constrained nonconvex quadratic problems was presented in references [13,15]. Quadratically constrained quadratic programs were considered in references [1,16,17]. An evolutionary approach was used to consider a class of standard quadratic optimization problems in Ref. [2]. However, differential quadratic programming problem such as the model (1.2) is seldom considered. Although it looks like an optimal control problem with r being the state and u being the control, there exist some differences between the DQP (1.2) and the optimal control problems. The main difference lies in the following aspect: the control u in DQP (1.2) is the solution at time t for DQP (1.2), whose coefficients depend on the current state, and it is a pointwise optimization, while the control u in the optimal control problems is to minimize a performance function that is an integral function.

Motivated by the work mentioned above, we consider the DQP (1.2) in this paper. The remainder of the paper is organized as follows: in the next section, some preliminary results are given. In Section 3, an existence theorem for the solution of the differential quadratic problem is proved under some suitable assumptions. In Section 4, an algorithm to compute the solution for differential quadratic problem is proposed and a convergence analysis for the algorithm is given. In Section 5, we provide some numerical examples to verify the validity of the algorithm. In the last section, the conclusion of this paper is given.

2. Preliminaries

For the convenience of studying, we restate the DQP (1.2) as the following differential inclusion problem,

$$\dot{r}(t) \in \mathbb{F}(r(t)), \quad r(0) = r^0, \tag{2.3}$$

where

$$\mathbb{F}(r(t)) = \{f(r(t), u(t)) : u(t) \in S(K, A(r(t)), h(r(t)), e(r(t)))\},$$

and $S(K, A(r(t)), h(r(t)), e(r(t)))$ denotes the solution set of the following quadratic programming problem,

$$\begin{array}{ll} \text{Minimize} & u(t)^T A(r(t))u(t) + h(r(t))^T u(t) + e(r(t)) \\ \text{subject to} & u(t) \in K. \end{array} \tag{2.4}$$

Definition 2.1. A pair $(r(t), u(t))$ defined on $[0, T]$ is called a Carathéodory weak solution of the differential quadratic programming problem (1.2) iff r is an absolutely continuous function on $[0, T]$ and satisfies the differential equation for almost all $t \in [0, T]$, and u is a measurable function such that

$$u(t) \in S(K, A(r(t)), h(r(t)), e(r(t)))$$

for almost all $t \in [0, T]$.

Definition 2.2. Let Y, Z be two topological spaces and $G : Y \rightrightarrows Z$ be a set-valued mapping with nonempty values. Then G is upper semicontinuous at $x_0 \in Y$ iff, for any neighborhood $\mathcal{N}(G(x_0))$ of $G(x_0)$, there exists a neighborhood $\mathcal{N}(x_0)$ of x_0 such that

$$G(x) \subset \mathcal{N}(G(x_0)), \quad \text{for every } x \in \mathcal{N}(x_0).$$

From Theorem 5.1 of [6], we get the following result easily.

Lemma 2.1. *Let $\mathbb{F} : \Omega \rightrightarrows R$ be an upper semicontinuous set-valued map with nonempty closed convex values, where $\Omega = [0, T] \times R$. If $\mathbb{F}(\Omega)$ is bounded, then for every $r^0 \in R$, the following differential inclusion problem,*

$$\dot{r} \in \mathbb{F}(t, r), \quad r(0) = r^0$$

has a weak solution in the sense of Carathéodory.

The following measurable selection lemma is a version of Filippov's implicit function theorem [7].