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Almost periodic sequence solution of a discrete Hassell–Varley predator-prey system with feedback control☆

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ABSTRACT

In this paper, we investigate a discrete Hassell–Varley response function predator-prey system with feedback control. First, sufficient conditions are established for the permanence of the system. Then, assuming that the coefficients of the system is almost periodic sequences, we obtain conditions for the existence and unique of the almost periodic solution. Moreover, the almost periodic solution we obtained is uniformly asymptotically stable.

1. Introduction

In 1969, Hassell and Varley proposed the following prey-predator model:

$$\begin{aligned} \frac{dx(t)}{dt} &= x(t)\left(r - bx(t) - \frac{\alpha y(t)}{my^{\gamma}(t) + x(t)}\right), \\ \frac{dy(t)}{dt} &= y(t)\left(-f + \frac{\beta x(t)}{my^{\gamma}(t) + x(t)}\right). \end{aligned} \tag{1.1}$$

The dynamic properties of the model such as the existence of positive solutions, the permanence of the system, and stability of the solutions have been intensively analyzed by numerous authors, see [1–4] and references cited therein. In 2011, Zhong and Liu [2] studied system (1.1). They obtained conditions for the permanence and extinction of the system and conditions of the existence and unique of periodic solution of the system with new estimate approach by using coincidence degree theorem, also they studied the existence of almost periodic solution of the system.

As was pointed out by many authors that the discrete time models governed by differential equations are more appropriate than the continuous ones when the populations have non-overlapping generations. Discrete time models can also provide efficient computational models of continuous models for numerical simulations. Indeed, much progress in the discrete time models have been made by many scholars, see, for examples, [3–5,7,9–11] and references cited therein. Recently, Wu and Li [4] studied

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the dynamic of the discrete predator-prey system with Hassell–Varley type functional response as follows:

$$\begin{aligned} x(k+1) &= x(k)\exp\left\{a(k)-b(k)x(k)-\frac{c(k)y(k)}{m(k)y^{\gamma}(k)+x(k)}\right\},\\ y(k+1) &= y(k)\exp\left\{-d(k)+\frac{f(k)x(k)}{m(k)y^{\gamma}(k)+x(k)}\right\}, \end{aligned} \tag{1.2}$$

where $x_i(k)(i=1,2)$ is the density of prey and predator species respectively at the kth generation; $a(k), b(k), c(k), m(k)$ stand for prey intrinsic growth rate, intra-specific competition, capturing rate, half saturation constant; $d(k), f(k)$ stand for predator death rate, maximal predator growth rate; $\frac{c(k)y(t)}{my^{\gamma}(t)+x(t)}, \frac{f(k)x(t)}{my^{\gamma}(t)+x(t)}$ denotes the Hassell–Varley functional response. By constructing a suitable Lyapunov function and using the comparison theorem of the difference equation, they got sufficient conditions which guarantees the permanence and global attractively of system (1.2).

On the other hand, ecosystem in the real world are continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables (see e.g. [6,7,10,11]). Though one can deliberately periodically fluctuate environmental parameters in controlled laboratory experiments, fluctuations in nature are hardly periodic. That is, almost periodicity is more likely to accurately describe natural fluctuations. Chen and Chen [10] obtained sufficient conditions for the existence of almost-periodic solution of delay population equation with feedback control. In Ref. [7], Niu and Chen discussed a discrete Lotka–Volterra competitive system with feedback control. Assuming that the coefficients of the system are almost periodic sequences, they obtained conditions for the existence and uniqueness of the almost periodic solution which is uniformly asymptotically stable. Li and Li [11] present a systematic analysis on the dynamics of feedback control discrete cooperation system. To the best of our knowledge, there are few works on the existence of almost periodic solution of the discrete system with Hassell–Varley functional response and feedback control. Motivated by the above consideration, we propose and study the following system:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{r(k)-b(k)x_1(k)-\frac{a_1(k)x_2(k)}{m(k)x_2^{\gamma}(k)+x_1(k)}-c_1(k)u_1(k)\right\},\\ x_2(k+1) &= x_2(k)\exp\left\{-d(k)+\frac{a_2(k)x_1(k)}{m(k)x_2^{\gamma}(k)+x_1(k)}-c_2(k)u_2(k)\right\},\\ u_1(k+1) &= (1-f_1(k))u_1(k)+e_1(k)x_1(k),\\ u_2(k+1) &= (1-f_2(k))u_2(k)+e_2(k)x_2(k), \end{aligned} \tag{1.3}$$

where $x_i(k)(i=1,2)$ is the density of prey and predator species respectively at the nth generation; $r(k)$ denotes the growth rate of prey and $u_i(k)(i=1,2)$ is the control variable; $\frac{a_i(k)x_j(k)}{m(k)x_2^{\gamma}+x_1(k)}(i,j=1,2,i\neq j)$ denotes the Hassell–Varley functional response; $\gamma\in(0,1)$ is constants. Assuming that all the parameters in model (1.3) is almost periodic sequence, we will discuss the existence and uniqueness of uniformly asymptotically stable almost periodic solutions of the system (1.3).

For each bounded sequence $\{r(k)\}$, we denote:

$$r^u=\sup_{k\in Z^+}\{r(k)\},\quad r^l=\inf_{k\in Z^+}\{r(k)\},\quad (r^u)^2=r^{u^2},\quad (r^l)^2=r^{l^2}.$$

Besides, we assume that

(H_0) $\{r(k)\}, \{b(k)\}, \{m(k)\}, \{d(k)\}, \{a_i(k)\}, \{c_i(k)\}, \{f_i(k)\}, \{e_i(k)\}, (i=1,2)$

are bounded non-negative almost periodic sequences such that

$$\begin{aligned} &0<r^l<r(k)<r^u, 0<b^l<b(k)<b^u, 0<m^l<m(k)<m^u, 0<d^l<d(k)<d^u,\\ &0<a_i^l<a_i(k)<a_i^u, 0<c_i^l<c_i(k)<c_i^u, 0<f_i^l<f_i(k)<f_i^u<1, 0<e_i^l<e_i(k)<e_i^u. \end{aligned}$$

Due to the biological interpretation of model (1.3), we assume that the initial condition of (1.3) satisfies

(H_1) $x_i(0)>0, u_i(0)>0, i=1,2.$

It is easy to see that the solutions of (1.3) with the initial condition (H_1) are defined and remain positive for all $k\in Z^+=\{0,1,2,3,\ldots\}$.

The main purpose of this paper is to derive a set of sufficient conditions for the permanence and the global existence of a unique positive almost periodic solution of (1.3). In Section 2, we recall some definitions and lemmas which are of importance in the proof of our main result. In Section 3, we establish sufficient conditions for system (1.3) to be permanent. In Section 4, we are concerned with some sufficient conditions which guarantee the existence and unique of the almost periodic solution. In addition, an example is given to illustrate our results obtained.