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Analysis and control of multiple chaotic attractors from a three-dimensional system

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ABSTRACT

This paper studies a three-dimensional multiple chaotic system with three quadratic nonlinear terms. The system is shown to exhibit multiple periodic attractors and multiple chaotic attractors including a four-scroll chaotic attractor. It is shown analytically and numerically that any neighborhood of the four-scroll chaotic attractor contains repelling sets with positive Lebesgue measures. Moreover, in terms of incommensurate fractional order systems, a simple fractional differentiator-based controller is designed to suppress chaos. Some basic dynamical behaviors of the system are investigated through analytical techniques and numerical simulation.

1. Introduction

In the last few decades, chaotic behavior and its control in dynamical systems have been extensively analyzed, along with the study of a rich class of dynamical behavior, including periodic oscillation, quasiperiodicity, bifurcation and control (see Refs. [1–25] and references therein). For example, in the study of periodic oscillation, the first harmonic method has been successfully applied by Aguirre et al. [1] and Suárez et al. [2]. Much work has been done in the analysis and identification of quasiperiodicity and bifurcation [3–8]. In particular, the basin of attraction and Hopf bifurcation were studied in refs. [9–11]. Also, exhibition of chaos has been extensively studied [12,13]. Furthermore, numerous methods for controlling Hopf bifurcation were presented by various authors [14–17,19–22]. However, the dynamics of multiple chaotic systems have not been thoroughly investigated until now, and some salient dynamical behavior such as Hopf bifurcation and chaotic behavior of three-dimensional multiple chaotic systems are still not well understood. The lack of understanding of multiple chaotic systems, despite their rich and fascinating behavior, has motivated the present work. The purpose of this paper is to study the properties of chaos and some subtle characteristics of Hopf bifurcation in new three-dimensional quadratic systems, and to reveal the geometrical structures of the lower dimensional multiple chaotic attractors. One of such systems has been studied by Guan et al. [5], which was inspired by the system proposed originally by Liu and Chen [6], i.e.,

$$\begin{cases} \dot{x} = ax - yz - y + k \\ \dot{y} = -by + xz \\ \dot{z} = -cz + xy \end{cases} \tag{1}$$

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where x, y, z are the state variables, $a, b, c > 0$ and k is a parameter. This three-dimensional autonomous smooth chaotic system has three quadratic nonlinear terms and one constant term.

Recently, due to their promising application in control, fractional order dynamic systems have been increasingly studied in the design and control of chaos. In [24], Mohammad et al. proposed a simple fractional differentiator-based controller to suppress chaos in a single input chaotic system. They applied the proposed controller to control chaos in a chaotic circuit which was introduced by Chen and Ueta [4]. Recently, based on (1), Guan et al. [5] studied numerically the complex dynamical behavior of multiple chaotic systems. In this paper, motivated by the works in [5,11,24], we consider the following system:

$$\begin{cases} \dot{x} = ax - yz - dy \\ \dot{y} = -by + xz \\ \dot{z} = -cz + xy \end{cases} \tag{2}$$

where $a, b, c, d > 0$, and c is chosen as the bifurcation parameter for the purpose of studying the basic dynamical properties. The chaotic attractors are first numerically verified through investigating phase trajectories, Lyapunov exponents, bifurcation paths, power spectra, and Poincaré projections. Moreover, by using a method similar to the one presented in [21,24], we propose and study a simple fractional differentiator-based controller, aiming to suppress chaos in this system.

As an initial quick glance, let us consider the following set of parameters for : $a = 3, b = 4, c = 0.075$ and $d = 0.45$. We can show that two chaotic attractors coexist for the system described by (2) with initial values (27.5, 1, 10) (red) and $(27.5, 1, -10)$ (blue), as shown in Fig. 1. The corresponding Lyapunov exponents of these two chaotic attractors are shown in Table 1. Specifically we see that the Lyapunov dimension of the first attractor is $D_L = j + (\sum_{i=1}^{j} \lambda_{LE_i})/|\lambda_{LE_{j+1}}| = 2.1176$. Accordingly, with a fractional Lyapunov dimension, the attractor exhibits chaotic behavior. Similarly, we can verify that the other attractor is also chaotic.

The key contributions of this paper can be highlighted as follows. We study a three-dimensional multiple chaotic system with three quadratic nonlinear terms. We show that the system exhibits multiple periodic attractors and multiple chaotic attractors including a four-scroll chaotic attractor. Existence and stability of the equilibrium of the system are studied, and a control method based on the stability consideration of the incommensurate fractional order system is derived to tame single input three-dimensional chaotic systems. We also show that any neighborhood of the four-scroll chaotic attractor contains repelling sets with positive Lebesgue measures.

The rest of this paper is organized as follows. In Section 2, the dynamical properties of the multiple chaotic system described by (2), including Lyapunov exponents, fractal dimension and chaotic behavior, are analyzed. It is shown that system (2) has coexisting chaotic and periodic attractors and can also generate a four-scroll chaotic attractor. In Section 3, a fractional differentiator-based controller for suppressing chaos is proposed. The tuning procedure for the proposed controller is developed on the basis of the stability consideration of the incommensurate fractional order system. Finally, Section 4 concludes the paper.

2. Analysis

In this section, we study the dynamical behavior of the system described by (2) via computer simulations. First, we analyze the evolution of chaotic attractors in this system through studying bifurcation diagrams, Lyapunov exponents, Poincaré mapping, power spectra and phase portraits. Second, we study the evolution of multiple attractors, and observe the intrinsic sensitivity to initial conditions. Finally, the four-scroll chaotic attractor from the system will be observed. Although much of the subsequent analysis can be repeated for a range of values of d. Notice that if $d = 0$, the system represented by (2) reduces to a chaotic system studied by Liu and Chen [6]. If $d = 1$, then (2) coincides with (1) with $k = 0$ and the results given in [5] correspond mainly to the case $k \neq 0$. Thus, the present paper focuses on the system represented by Eq. (2). For the purpose of illustration, we will simplify the presentation by choosing $d = 1$.

2.1. Dissipativity of the new system

For the system described by (2), we first note that

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -(b + c - a) \tag{3}$$

where V is the volume element of the flow. Obviously, when $b + c > a$, the system has a dissipative structure, with an exponential contraction rate given by $\frac{dV}{dt} = -(b + c - a)V$. In other words, a volume element V_0 is contracted by the flow into a volume element $V_0 e^{-(b+c-a)t}$ in time t. Thus, each volume containing the system orbit shrinks to zero as $t \to \infty$ at an exponential rate given by $-(b + c - a)$, which is independent of x, y, z. As a result, all system orbits are finally confined to some subset of zero volume, and the asymptotic motion settles onto an attractor in the three-dimensional phase plane.