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# Further results on exponential stability for impulsive switched nonlinear time-delay systems with delayed impulse effects

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ABSTRACT

This paper is concerned with the problem of exponential stability for a class of impulsive switched nonlinear time-delay systems with delayed impulse effects. By using the multiple Lyapunov–Krasovskii functionals technique, some exponential stability criteria are obtained, respectively, for two kinds of impulsive signals (destabilizing impulsive signal and stabilizing impulsive signal). The derived results not only characterise the effects of delayed impulse, time delay and switching on nonlinear systems, but also remove some restriction conditions. Furthermore, a more precise bound of system state is given. Compared with existing results on related problems, the obtained results are less conservative. Three examples are provided to illustrate the effectiveness and the generality of the proposed results.



## 1. Introduction

In the past decades, the problems of stability and impulsive stabilization for impulsive differential systems have received considerable research attention [1–11]. In [12–19,26,27], the exponential stability results for impulsive functional differential systems are established. In most of the literatures, the impulses which represent abrupt changes of system state at instantaneous time $t_k$, usually take the form $\triangle x(t_k) := x(t_k^+) - x(t_k^-) = B_k x(t_k^-)$ [11,20]. However, input delays are often encountered in transmitting the impulse information. For example, for networked control systems, the output is transmitted through a digital communication network. Because the computation and network itself result in time delays, the $k$th input update time $t_k$ is greater than the $k$th sampling time. Via choosing the impulsive control law $\Delta x(t_k) = B_k u_k$ with $u_k = K_k y_k$, we can derive the delayed impulses $\Delta x(t) = B_k x((t-d_k)^-)$ at $t = t_k$ [21]. We will consider more general impulses taking the form $x(t^+) = C_{0k} x(t^-) + C_{1k} x((t-d_k)^-)$ at $t = t_k$ in the present paper.

Switched systems can be described by a differential equation whose right-hand side is chosen from a family of functions according to switching signals. Although switched systems are important models for dealing with many complex physical processes, they do not cover those phenomena displaying certain kinds of dynamics with impulses. Therefore, a more comprehensive model – impulsive switched system – has been introduced to describe both the impulse effects and the switching effects. And the switching law and the impulsive signal can be naturally integrated as an impulsive and switching law. In recent years, impulsive switched systems have received only moderate attention despite of the apparent abundance of applications (see, e.g., [23–25,28,29]).

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As a powerful tool to analyze the stability of impulsive and switched systems, Lyapunov function technique usually is classified into two classes, that is, the Lyapunov–Razumikhin function method [1,10,13,18,19] and the Lyapunov–Krasovskii functional method [9,22]. For instance, in [9], Liu investigates the exponential stability of impulsive systems with time delay by Lyapunov–Krasovskii functionals technique. In [1], using Lyapunov–Razumikhin approach, the exponential stability for nonlinear time-delay systems with delayed impulses are derived, respectively, for two kinds of delayed impulses. However, the impacts of switching signals may be neglected. Further, in [17] the exponential stability problem is considered for a class of nonlinear impulsive and switched time-delay systems with delayed impulse by employing multiple Lyapunov–Krasovskii functionals technique. Nevertheless, those conditions in [17] have a restriction of $p_1 \geq p_2 \geq 1$ and the parameter $p_2$ is not involved, which may lead to conservatism in some sense. In [9], the exponential stability of impulsive systems with time delay is studied using Lyapunov–Krasovskii functional method. But a constraint is imposed on the impulse frequency with a parameter $\kappa_i$. In this paper, a more general impulsive switched time delay system with delayed impulses is investigated which include the system in [9] as a special case. It is worthy to consider the impacts of the delayed impulses, switching signals, and the parameter $p_2$ on the performance of nonlinear systems. Such an open yet challenging issue arouses our initial study on the exponential stability of impulsive switched nonlinear time delay systems with delayed impulses.

In this paper, we address the exponential stability of nonlinear time-delay system with more general delayed impulses via employing Lyapunov–Krasovskii functional technique. The main contributions of this paper can be strengthen as follows. (1) This paper considers switching signal, impulsive disturbance, time delay and delayed impulse effects in the meantime, which makes it hard to investigate the exponential stability but worthwhile in the sense that it may bring much attention to this field. (2) The condition that each impulse is required to stabilize the system existing in most of the literatures may be relaxed. The constants $q_k$ is introduced such that it is not strictly required that each impulse contributes to stabilize the system, as long as the overall contribution of the impulses are stabilizing. (3) Compared with [1,9,17], the requirement of $p_2 \geq 1$ and the limitation of impulse frequency with a parameter $\kappa_i$ are removed. A more accurate bound of system state is given by the parameter $p_2$. Thus, the results obtained in this paper is less conservative than [1,9,17]. In particular, for each type of impulses (stabilizing and destabilizing delayed impulses), we provide two classes of stability conditions in terms of three cases for exponential stability. Finally, three numerical simulations are provided to demonstrate the effectiveness of the proposed results.

## 2. Problem formulation and preliminaries

Let $N$ denote the set of positive integers, $R^+$ denote the set of non-negative real numbers, and $R^n$ the $n$-dimensional real Euclidean space. $|\cdot|$ denotes the Euclidean norm for vectors or the spectral norm for matrices. Define $\phi(t^+) = \lim_{s\to t^+} \phi(s)$, and $\phi(t^-) = \lim_{s\to t^-} \phi(s)$. For $\tau > 0$, let $PC([-\tau, 0], R^n)$ denote the class of functions $\phi : (-\tau, 0] \to R^n$ satisfying the following: $\phi(t) = \phi(t^+)$, $\forall t \in [-\tau, 0)$; Also, $\phi(t) = \phi(t^-)$ for all but at most a finite number of points $t \in (-\tau, 0]$, and the norm is defined by $\|\phi\|_\tau = \sup_{-\tau\leq s\leq 0} \|\phi\|$. Given $x \in PC([t_0 - \tau, +\infty], R^n)$ and for each $t \geq t_0$, define $x_t, x_{t^-} \in PC([t_0 - \tau, +\infty], R^n)$ by $x_t(s) = x(t+s)$ for $-\tau \leq s \leq 0$ and $x_{t^-}(s) = x(t+s)$ for $-\tau \leq s < 0$, respectively. For a given scalar $\rho \geq 0$, let $B(\rho) = \{x \in R^n : |x| \leq \rho\}$.

A function $\alpha : R^+ \to R^+$ is said to be of class $\mathcal{K}$ and we write $\alpha \in \mathcal{K}$, if $\alpha$ is continuous and increasing strictly, and $\alpha(0) = 0$; if it also satisfies $\alpha(t) \to \infty$ as $t \to \infty$, we say that $\alpha$ is of class $\mathcal{K}_\infty$ and we write $\alpha \in \mathcal{K}_\infty$.

Let $\mathcal{N}_c, \mathcal{N}_d$ be arbitrary index sets. Consider the following impulsive switched nonlinear time-delay system with delayed impulses

$$\begin{cases} \dot{x} = f_{i_k}(t, x_t), \quad t > t_0, \quad t \neq t_k, \quad i_k \in N_c, \quad k \in N, \\ x(t) = g_{j_k}(x(t^-), \quad x((t-d_k)^-)), \quad t = t_k, \quad j_k \in N_d, \quad k \in N \setminus \{0\}, \\ x(t_0 + \theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $\dot{x}(t)$ is the right-hand derivative of $x(t)$, $\{t_k : k \in N\} \subset R^+$ is a strictly increasing sequence and $\lim_{k\to\infty} t_k = \infty$. $\{d_k \geq 0 \colon k \in N\}$ are the impulse input delays with $\max_k d_k \leq d < \infty$. For each $i \in \mathcal{N}_c$ and $j \in \mathcal{N}_d$, we assume that $f_i : R^+ \times PC \to R^n$ and $g_j : R^+ \times PC \to R^n$ satisfy $f_i(t, 0) \equiv g_j(t, 0) \equiv 0$. Moreover, we make the following assumptions on systems (1).

($\mathbf{A_1}$) $f_i(t, \psi)$ is composite-PC, i.e., for each $t_0 \in R^+$ and $\sigma > 0$, if $x \in PC([t_0 - \tau, t_0 + \sigma], R^n)$ and $x$ is continuous at each $t \neq t_k$ in $(t_0, t_0 + \sigma]$, then the composite function $f_i(t, \psi)$ is element of the function class $PC([t_0, t_0 + \sigma], R^n)$.

($\mathbf{A_2}$) $f_i(t, \psi)$ is quasi-bounded, i.e., for each $t_0 \geq 0$ and $\sigma > 0$, and for each compact set $F \subset R^n$, there exists some $M_i > 0$ such that $|f_i(t, \psi)| \leq M_i$ for all $i \in \mathcal{N}_c$ and $(t, \psi) \in [t_0, t_0 + \sigma] \times PC([-\tau, 0], F) \times R^m$.

($\mathbf{A_3}$) For each fixed $t \in R^+$, $f_i(t, \psi)$ is a continuous function of $\psi$ on $PC([-\tau, 0], R^n)$.

($\mathbf{A_4}$) There exist scalars $K_1^i > 0$, and class $\mathcal{K}_\infty$ function $\chi$ such that $|f_i(t, \psi)| \leq K_1^i \|\psi\|_\tau$, for any $\psi \in PC([-\tau, 0], B(\rho))$. Set $K_1 = \sup_{i\in\mathcal{N}_c} K_1^i$.

($\mathbf{A_5}$) There exist non-negative bounded scalar sequences $\{h_{0k}\}, \{h_{1k}\}$ such that $|g_{j_k}(x, y) - x| \leq h_{0k}|x| + h_{1k}|y|$, $\bar{h} = \sup_k\{h_{0k} + h_{1k}\}$.

($\mathbf{A_6}$) There exist scalars $K_2^i > 0$, such that $|g_{j_k}(x, y_1) - g_{j_k}(x, y_2)| \leq K_2|y_1 - y_2|$ for every $k \in N$ and $y_1, y_2 \in B(\rho)$.

In [6], the existence and uniqueness results are established for impulsive delay systems without switching. The case for system (1) with switching is essentially the same, by an argument using the method of steps over all the switching/impulse intervals. So, according to Theorems 3.1 and 3.2 in [6], it was shown that under assumptions $(A_1) - (A_3)$, system (1) admits a solution $x(t, t_0, \phi)$ for any $\phi \in PC([-\tau, 0], R^n)$ which exists in a maximal interval $[t_0 - \tau, t_0 + b)$ where $0 < b \leq \infty$.