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The algorithm for the optimal cycle time and pricing decisions for an integrated inventory system with order-size dependent trade credit in supply chain management

Kun-Jen Chung [a,b,c], Jui-Jung Liao [d], Pin-Shou Ting [c], Shy-Der Lin [e], H.M. Srivastava [f,*]

[a] Chung Yuan Christian University, Chung-Li 32023, Taiwan, ROC
[b] National Taiwan University of Science and Technology, Taipei 10607, Taiwan, ROC
[c] Department of International Business Management, Shih Chien University, Taipei 10462, Taiwan, ROC
[d] Department of Business Administration, Chihlee Institute of Technology, Taipei 22050, Taiwan, ROC
[e] Departments of Applied Mathematics and Business Administration, Chung Yuan Christian University, Chung-Li 32023, Taiwan, ROC
[f] Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3R4, Canada

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ABSTRACT

A given inventory problem consists of two parts: (1) the modeling part and (2) the solution procedure part. The modeling part can provide insight into the solution of the inventory problem and the solution procedure part involves the implementation of the inventory model. Both the modeling part and the solution procedure part of the inventory problem are equally important. Recently, Ouyang et al. [17] developed an integrated inventory model with a price-sensitive demand rate and determined both the economic lot size of the buyer's ordering and the supplier's production batch in order to maximize the total profit per unit time. Basically, their modeling is correct and interesting. They developed an algorithm based upon the first-order condition and the second-order condition to locate the optimal solution. However, the fundamentals of mathematics and the numerical examples which are considered in this paper illustrate that their algorithm based upon the first-order condition and the second-order condition to locate the optimal solution has several shortcomings. These shortcomings are shown here to influence the accuracy of the implementation of the inventory model. Since there exist reasons and motivations to present the correct solution procedures to the targeted readers, the main purpose of this paper is to adopt the rigorous methods of mathematical analysis in order to develop the complete solution procedures to locate the optimal solution for removing shortcomings in the earlier investigation by Ouyang et al. [17].

1. Introduction and motivation

A large number of articles dealing with inventory models focus only on determining the optimal policy for the retailer or the supplier. However, in practice, many purchasing situations reveal that different considerations from both the retailer and the supplier frequently conflict with one another. In this connection, some recent investigations by (for example) Chung et al. [5] to [9] may be cited. Goyal [12] was probably the first researcher to develop the seller–customer integrated inventory model. In

* Corresponding author. Tel.: +1 250 472 5313; fax: +1 250 721 8962.
E-mail addresses: kunjenchung@gmail.com (K.-J. Chung), liaojj@mail.chihlee.edu.tw (J.-J. Liao), ting@mail.usc.edu.tw (P.-S. Ting), shyder@cycu.edu.tw (S.-D. Lin), harimsri@math.uvic.ca (H.M. Srivastava).

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general, the integrated models usually have the advantage of reducing the total cost. In the modern global competitive market, the supplier and the retailer should be treated as strategic partners in the supply chain with a long-term cooperative relationship. Goyal [11] extended the lot-for-lot policy discussed in Goyal [13] and illustrated the fact that the inventory cost could be significantly reduced if the supplier's economic production quantity was an integer multiple of the retailer's purchase quantity. Lu [16] further generalized the work of Goyal [13] by relaxing the assumption that the supplier could supply the retailer only after completing the entire lot size. On the other hand, in the real world, suppliers usually provide a delay period in payment to encourage retailers to buy more of the ordered quantity. During the trade credit period, the retailers can obtain the interest from the nonpayment and sales income, while suppliers lose the interest income during the same time. Goyal [12] developed the EOQ (economic order quantity) model under conditions of permissible delay in payments. Chen and Kang [1] first incorporated the works by Goyal (see [11] and [12]) to establish some integrated vendor–buyer cooperative inventory models with permissible delay in payments. In addition, inventory models involving trade credit usually make the assumption that the credit period offered by the supplier is constant. However, in practice, the length of the credit period might be adjusted with the order quantities and much attention has been paid recently to this fact. Shinn and Hwang [20] assumed that an order-size dependent trade credit is offered by the supplier. In their model, once a purchasing volume breakpoint is exceeded, a certain credit period is offered to the buyer. A longer credit period is offered for a larger amount of purchase. Ouyang et al. [17] incorporated the aforementioned earlier works by Goyal [11], Lu [16] and Shinn and Hwang [20] to establish an integrated inventory model to determine how the order decisions (timing and size), the delivery decision (shipments from the supplier to the buyer in one production run) and the pricing decision should be made in order to maximize the joint total profit per unit time. Basically, their inventory problem is rather interesting meaningful. They developed an algorithm based upon the first-order condition and the second-order condition to locate the optimal solution. However, the fundamentals of mathematics and the numerical examples (which we propose to consider in this paper) illustrate the fact that their algorithm based upon the first-order condition and the second-order condition to locate the optimal solution has several shortcomings. These shortcomings are shown to influence the accuracy of the implementation of the inventory model. Furthermore, since there exist reasons and motivations to present the correct solutions to readers, the main purpose of this paper is to adopt the rigorous methods of mathematical analysis in order to develop the complete solution procedures with a view to locating the optimal solution by appropriately removing the shortcomings in the work of Ouyang et al. [17]. Several other interesting developments on the subject of our present investigation can be found in a number of related recent works (see, for example, [2], [3], [6–8], [10], [14], [15], [18], [19], [21] and [24]).

2. The mathematical model

The following assumptions and notations are adopted to develop the proposed model:

1. The discussion and analysis in this paper is restricted to the case of a single-supplier and single-buyer of a specific product.
2. Shortages are not permitted and the time horizon over which the product is ordered by the buyer and supplied by the supplier is infinite.
3. The demand for the product is assumed to be retail-price sensitive and is given by $D(p) = ap^{-\delta}$, where $a > 0$ is a scaling factor, and $\delta > 1$ is a price-elasticity coefficient. For notational simplicity, $D(p)$ and D will be used interchangeably in this paper.
4. For the buyer, the inventory cycle length is T and order quantity is $Q\ (=DT)$ per order. The buyer incurs an ordering cost, S_B, which includes the administrative costs of placing, tracking, shipping, receiving and paying for each order.
5. During the production period, the supplier manufactures in batches of size nQ where n is an integer and incurs a batch setup cost S_V. Once the first Q units are produced, the supplier delivers them to the buyer and then continuous making the delivery on average every $T(=Q/D)$ units of time until the supplier's inventory level falls to zero.
6. The capacity utilization, ρ, is the ratio of the demand rate, D, to the production rate, R, which is given and is less than 1, that is, $\rho = D/R$ and $\rho < 1$.
7. For each unit of product, the supplier spends \$$c$ in production and receives \$$v$ from the buyer. After that, the buyer sells it by \$$p$ to his/her customers. The relationship among them is $p > v > c$.
8. The carrying cost rate, excluding interest charge for the supplier, is r_V and r_B for the buyer.
9. The credit period $M_j\ (j = 1, 2, \ldots, k)$, which is offered by the supplier, is related to the buyer's order quantity where the higher order quantity the longer the credit period, and is given as follows:

$$M = \begin{cases} M_1 & q_1 \leqq Q < q_2, \\ M_2 & q_2 \leqq Q < q_3, \\ \vdots & \\ M_k & q_k \leqq Q < q_{k+1}, \end{cases}$$

where

$$1 \leqq q_1 < q_2 < \cdots < q_{k+1} = \infty$$

is the sequence of quantities at which a specific credit period is offered. M_j denotes the trade credit applicable to lot size falls in the interval q_j to q_{j+1} with

$$0 < M_1 < M_2 < \cdots < M_k.$$