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Asymptotic behavior of switched delay systems with nonlinear disturbances

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ABSTRACT

The objective of this paper is to study the asymptotic behavior of switched delay systems with nonlinear disturbances. By establishing a new delay Gronwall–Bellman integral inequality and an elementary inequality, we obtain some asymptotic results for the systems under arbitrary switching laws, which extend some existing results in the literature to the more general nonlinear case. Finally, two examples are provided to illustrate the effectiveness of our results.

1. Introduction

A switched system which consists of a series of dynamical subsystems and a switching signal is a type of hybrid dynamical system. The theory of switched systems has historically assumed a position of great importance in systems theory due to their strong engineering backgrounds (e.g. [1,2]), and has been studied extensively in recent years [3–17].

One fundamental research issue for switched systems is stability. For the switched linear systems under arbitrary switching, common quadratic Lyapunov functions (or switched quadratic Lyapunov functions) were employed to investigate the asymptotic stability (see [7]). In [12], Alwan and Liu studied the linear and weakly nonlinear switched delay systems and obtained several exponential stability criteria by using multiple Lyapunov functions technique and dwell-time approach. Some necessary and sufficient conditions were also provided to ensure the asymptotic stability of switched linear systems under arbitrary switching laws [8]. For the switched delay system under restricted switching, the authors used the average dwell time approach to study the stability [3–5,10]. Another method commonly used to study the switched systems was based on Gronwall-type inequalities. For example, these inequalities have been applied to study the stabilization of switched linear systems with nonlinear impulse effects and disturbances in [9], to analyze the stabilization of switched delay systems in [14], and to study the finite-time stability for nonlinear impulsive switched systems in [11]. Some generalizations of Gronwall–Bellman inequalities applied to switched linear systems can be found in [6].

Motivated by the works [6,10,12], we investigate the asymptotic behavior of the switched delay systems with nonlinear disturbances. The main contributions of this paper can be summarized as follows: (1) a new delay Gronwall–Bellman inequality and an elementary inequality are established which play a key role in proving the main theorems; (2) compared with the system studied in [6], we study the general system which allows the distances may have the nonlinear delay terms. Consequently, the main results of this paper generalize the results given in [6].

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The rest of this paper is organized as follows. In Section 2, a switched delay system with nonlinear disturbances is introduced and some useful definitions and lemmas are also presented. We state and prove our main results in Section 3. In Section 4, two examples are provided to illustrate the main results. Finally, Section 5 gives a conclusion.

Notation. Throughout the paper, R denotes the set of real numbers, $R_+ = [0, +\infty)$, R^n denotes the n-dimensional Euclidean space with vector norm $\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$. For $A \in R^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are the maximum and minimum eigenvalues of A, respectively. $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$.

2. Problem description and preliminaries

Consider the following switched delay system with nonlinear disturbances:

$$\begin{cases} x'(t) = A_{\sigma(t)}x(t) + f_{\sigma(t)}(t, x(t), x(t-h(t))), & t \geq 0, \\ x(t) = \varphi(t), & t \in [-h_1, 0], \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $\sigma(t) : [0, +\infty) \to \Lambda$ is a piecewise constant function, $\Lambda = \{1, 2, \ldots, N\}$, and N is the number of subsystems. For a switching sequence $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\sigma(t)$ is everywhere continuous from the right. When $t \in [t_k, t_{k+1})$, we say the $\sigma(t_k)$th subsystem is active. $h(t)$ is a time-varying delay satisfying $0 \leq h(t) \leq h_1$. $\varphi(t) \in C([-h_1, 0], R^n)$, where $C([-h_1, 0], R^n)$ is the Banach space of all continuous functions on $[-h_1, 0]$ with values in R^n normed by the maximum norm $\|\varphi\|_{h_1} = \max_{t\in[-h_1,0]} \|\varphi(t)\|$. For any $i \in \Lambda$, the disturbance $f_i(t, x, y) : R_+ \times R^n \times R^n \to R^n$ is a continuous function, which satisfies local Lipschitz condition with respect to x, y. The solution of system (2.1) is said to be well defined if for any switching signal $\sigma(t)$ and initial condition $x(t) = \varphi(t), t \in [-h_1, 0]$, there exists a unique absolutely continuous function $x(t)$ on $[-h_1, +\infty)$ such that $x(t)$ is a solution of system (2.1). Throughout this paper, we assume that the local Lipchitz condition holds for the subsystems, and thus the well-definedness of the switched system is guaranteed. Furthermore, we assume the following hypotheses:

(H_1) $A_i, i \in \Lambda$, are Hurwitz matrices. That is, there exist two positive constants M and λ such that $\|e^{A_i t}\| \leq Me^{-\lambda t}$ for $i \in \Lambda$;
(H_2) $A_iA_j = A_jA_i$ for $i, j \in \Lambda$;
(H_3) $f_i(t, x, y), i \in \Lambda$, satisfy $\|f_i(t, x, y)\| \leq \varepsilon_1\|x\| + \varepsilon_2\|y\| + \varepsilon_3\|x\|^p + \varepsilon_4\|y\|^q + \alpha(t)$, where p, q are constants satisfying $p, q > 0$ and $p, q \neq 1$, $\varepsilon_i \geq 0, i = 1, 2, 3, 4$, are constants, and $\alpha(t)$ satisfies $\int_0^{+\infty} e^{\lambda t}\alpha(t)dt < \infty$.

Remark 1. In [6], the author assumed that the nonlinear terms satisfy $\|f_i(t, x)\| \leq \varepsilon_1\|x\| + \varepsilon_2\|x\|^p + \alpha(t)$. It is obvious that the assumption on the nonlinear terms given here is more general. In addition, the existence of the nonlinear delay terms leads to some difficulty when considering the asymptotic behavior of the system. To overcome this difficulty, we introduce an elementary inequality and a new delay Gronwall–Bellman integral inequality.

The following definitions and lemmas are very useful in the proof of our main results.

Definition 2.1. The solution of system (2.1) is said to be asymptotically convergent to zero with decaying rate λ if there exist positive constants k and λ such that

$$\|x(t)\| \leq ke^{-\lambda t}, \quad t \geq 0.$$

Definition 2.2. The solution of system (2.1) is bounded if there exists a positive constant M such that

$$\|x(t)\| \leq M, \quad t \geq 0.$$

Lemma 2.1. *Let u be a nonnegative function, $p > q > 0, c_1 \geq 0$, and $k_1 > 0$. Then, we have*

$$c_1u^q \leq k_1u^p + \theta(p, q, c_1, k_1),$$

where

$$\theta(p, q, c_1, k_1) = \left(1 - \frac{q}{p}\right)\left(\frac{q}{p}\right)^{\frac{q}{p-q}} c_1^{\frac{p}{p-q}} k_1^{\frac{q}{q-p}}.$$

Proof. Set $F(u) = c_1u^q - k_1u^p$. It is seen that $F(u)$ obtains its maximum at $u = (qc_1/(k_1p))^{1/(p-q)}$ and

$$F_{\max} = \left(1 - \frac{q}{p}\right)\left(\frac{q}{p}\right)^{\frac{q}{p-q}} c_1^{\frac{p}{p-q}} k_1^{\frac{q}{q-p}}.$$

The proof is complete. □

Lemma 2.2. *Assume that k and p are two constants satisfying $k \geq 0, p > 0$, and $p \neq 1$, $u(t), f(t), g(t), h(t), l(t) \in C(R_+, R_+)$, and $u(t)$ satisfies the following delay integral inequality*

$$u(t) \leq k + \int_0^t (f(s)u(s) + g(s)u(\tau(s)) + h(s)u^p(s) + l(s)u^p(\tau(s)))ds$$