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Successive approximation of neutral functional stochastic differential equations with variable delays[☆]

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ABSTRACT

By using successive approximation, we prove the existence and uniqueness of initial value problem for stochastic differential equations driven by both the cylindrical Brownian motion and by the variable delays in a Hilbert space with non-Lipschitzian coefficients. Moreover, the numerical solutions are shown to converge uniformly to the analytical solutions of the stochastic differential equation.

1. Introduction

Neutral functional stochastic differential equation (NFSDE) is a class of equations depending on past as well as present values, but which involve derivatives with delays as well as the function itself. NFSDEs are not only an extension of functional differential equations, but also provide good models in many fields including biology, electronics, mechanics and economics. The theory of NFSDE in finite-dimensional spaces has been extensively studied in the literature, see Kolmanovskii and Nosov [1], Mao [2], Kolmanovskii et al. [3] and Liu and Xia [4] and references therein.

Recently, in the infinite-dimensional Hilbert space, some new results have been obtained in this field despite the importance and interest of the NFSDEs. In 2010, Brahim and Salah [5] studied the following neutral functional stochastic differential equations driven both by the cylindrical Brownian motion and by the Poisson point processes:

$$\begin{aligned} d[x(t)+g(t,x(t-r))] &= [Ax(t)+f(t,x_t)]dt+\sigma(t,x_t)dW(t) \\ &\quad + \int_{\mathcal{U}} h(t,x_t,u)\tilde{N}(dt,du),\ 0\le t\le T, \\ x(t) &= \phi(t),\ -\tau\le t\le 0, \end{aligned} \tag{1.1}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators. By using successive approximation, the authors obtained some existence results for (1.1). To be more precise, a version of (1.1), when the delays are constant and $h = 0$, the authors [6] studied the above equation and obtained some stability properties of the mild solutions. In [7], the existence and uniqueness of mild solutions to model (1.1) with $h = 0$ is studied, as well as some results on the stability of the null solution. In [8] the authors studied the problem in a variational point of view.

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Motivated by the above work, the purpose of this paper is to prove the existence and uniqueness of mild solutions for a class of NFSDE with variable delays described in the form

$$\begin{aligned} d[x(t)+cx(t-\tau)] &= [Ax(t)+f(t,x(t-\delta(t)))]dt+\sigma(t,x(t-\delta(t)))dW(t),\ 0\le t\le T, \\ x(t) &= \phi(t),\ -\tau\le t\le 0, \end{aligned} \tag{1.2}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators, $(T(t))_{t\ge 0}$, in a Hilbert space H; c and $\tau>0$ are given constants; $\delta(t)$: $[0, T]\to[0,\tau]$ is one-order differentiable function; $f:[0,T]\times\mathcal{D}_\tau\to H$; $\sigma:[0,T]\times\mathcal{D}_\tau\to\mathcal{L}_2(Q^{1/2}K,H)$ are appropriate functions. Here $\mathcal{L}_2(Q^{1/2}K,H)$ denotes the space of all Q-Hilbert–Schmidt operators from $Q^{1/2}K$ into H.

2. Preliminaries

In this section, we introduce notations, definitions and preliminary results which we require to establish the existence and uniqueness of a solution of Eq. (1.2).

Let $(\Omega,\mathcal{F},\mathbb{P})$ be a complete probability space equipped with some filtration $(\mathcal{F}_t)_{t\ge0}$ satisfying the usual conditions, i.e., the filtration is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets. Let K and H be two real separable Hilbert spaces and denote by $L(K, H)$ the family of bounded linear operators from K to H. Fix a non-negative and symmetric operator $Q\in L(K,K)$ and let Q be a cylindrical Q-Wiener process in K defined on the filtered probability space $(\Omega,\mathcal{F},\mathbb{P},(\mathcal{F}_t)_{t\ge0})$. Denote by $\mathcal{L}_2:\mathcal{L}_2(Q^{1/2}K,H)$ the space of all Q-Hilbert–Schmidt operators from $Q^{1/2}K$ into H with the inner product $\langle\phi,\varphi\rangle=\mathrm{tr}[\phi Q\varphi]$.

Giving two fixed positive constants T and τ, we denote by $\mathcal{D}_\tau:=D([-\tau,0];H)$ the space of all functions from $[-\tau,0]$ into H that are right continuous and have left limits at every point (càdlàg for short), endowed with the sup norm. Obviously, $\forall t\in[0,T]$, $x(t-\delta(t))=x_t(-\delta(t))\in D_\tau$. Let the Banach space B_T of all H-valued $\mathcal{F}_t$ adapted process $x(t,\omega):[-\tau,T]\times\Omega\to H$, which are càdlàg in t for a.e. fixed $\omega\in\Omega$ and satisfy

$$||x||^2_{B_T}=E\sup_{-\tau\le t}||x(t,\omega)||^2<\infty.$$

Let $\mathcal{A}$ denote the cylindrical σ-algebra generated by all point evaluations $Z\to Z(s)$ for $z\in\mathcal{D}_\tau$ and arbitrary $s\in[-\tau,0]$ and denote by $\mathcal{M}_T$ and $\mathcal{P}_T$ the σ-algebra of progressive measurable subsets and predictable subsets of $[0, T]\times\Omega$ respectively. Define

$$\mathcal{F}^2([0,T],H)=\{f:[0,T]\times\Omega\to H/f\text{ is }\mathcal{M}_T/\mathcal{B}(H)\text{ measurable and }\int_0^T E||f(t,\omega)||^2_H<\infty\},$$

$$\mathcal{F}^2([0,T],\mathcal{L}_2)=\{f:[0,T]\times\Omega\to\mathcal{L}_2/f\text{ is }\mathcal{M}_T/\mathcal{B}(\mathcal{L}_2)\text{ measurable and }\int_0^T E||f(t,\omega)||^2_{\mathcal{L}_2}<\infty\},$$

here $\mathcal{B}(Y)$ denotes the Borel σ-field on a topological space Y. Obviously they are Hilbert spaces with respect to the norm

$$||f||^2_{\mathcal{F}^2([0,T],H)}=\int_0^T E||f(t,\omega)||^2_H dt,\ ||f||^2_{\mathcal{F}^2([0,T],\mathcal{L}_2)}=\int_0^T E||f(t,\omega)||^2_{\mathcal{L}_2}dt.$$

Let A: $D(A)\to H$ be the infinitesimal generator of an analytic semigroup, $(T(t))_{t\ge0}$, of bounded linear operators on H. It is well known that there exist $M\ge1$ and $\lambda\in\mathbb{R}$ such that $||T(t)||\le Me^{\lambda t}$ for every $t\ge0$.

If $(T(t))_{t\ge0}$ is a uniformly bounded and analytic semigroup such that $0\in\rho(A)$, where $\rho(A)$ is the resolvent set of A, then it is possible to define the fractional power $(-A)^\alpha$ for $0<\alpha\le1$, as a closed linear operator on its domain $D(-A)^\alpha$. Furthermore, the subspace $D(-A)^\alpha$ is dense in H, and the express endowed

$$||h||_\alpha=||(-A)^\alpha h||$$

defines a norm in $D(-A)^\alpha$. If H_α represents the space $D(-A)^\alpha$ endowed with the norm $||\cdot||_\alpha$, we have the following lemma:

Lemma 2.1 ([9]). *Suppose that the following conditions are satisfied.*

(1) *Let $0<\alpha\le1$. Then H_α is a Banach space.*
(2) *If $0<\beta\le\alpha$ then the injection $H_\alpha\hookrightarrow H_\beta$ is continuous.*
(3) *For each $0<\alpha\le1$ there exists $C_\alpha>0$ such that*

$$||(-A)^\alpha T(t)||\le\frac{C_\alpha}{t^\alpha},\ 0<t\le T.$$

Now, we give the following Bihari's inequality and Burkholder's inequality:

Lemma 2.2 ([10]). *Let $\rho:\mathbb{R}^+\to\mathbb{R}^+$, $\mathbb{R}^+=[0,+\infty)$ be a continuous and non-decreasing function and let g, h, λ be non-negative functions on $\mathbb{R}^+$ such that*

$$g(t)\le h(t)+\int_0^t\lambda(s)\rho(g(s))ds,\ t\ge0,$$

then

$$g(t)\le G^{-1}(G(h^*(t))+\int_0^t\lambda(s)ds),$$