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Inverse problem for interior spectral data of discontinuous Dirac operator

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ABSTRACT

In this work, an inverse problem for Dirac operator with discontinuities is studied. It is shown that the potential functions can be uniquely determined by a set of values of eigenfunctions at some interior point and parts of two spectra.

1. Introduction

We consider the canonical form of the Dirac operator $L(Q(x); a, \alpha, \beta)$, generated by the differential equations

$$ly := By\prime + Q(x)y = \lambda y, \quad 0 < x < 1, \tag{1.1}$$

with

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ Q(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

subject to the boundary conditions

$$\begin{cases} U(y) & := y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0, \\ V(y) & := y_1(1)\cos\beta + y_2(1)\sin\beta = 0, \end{cases} \tag{1.2}$$

and discontinuity conditions

$$y\left(\frac{1}{2} + 0\right) = Ay\left(\frac{1}{2} - 0\right), \tag{1.3}$$

where λ is a spectral parameter, $p(x)$ and $q(x)$ are real-valued functions in $L^2(0, 1)$, $0 \le \alpha, \beta < \pi$, $A = \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$, $a \in \mathbb{R}^+/\{1\}$.

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The basic and comprehensive results about Dirac operators were given in [8]. Furthermore, spectral problems for Sturm–Liouville or Dirac operators were extensively studied in various publications, see e.g. [2–4,9,10,14,16,17].

Boundary value problems with discontinuous inside the interval often appear in mathematics, mechanics, physics, geophysics and other branches of natural properties. Ref. [5] is well known work about discontinuous inverse eigenvalue problems. Direct and inverse problems for Dirac operators with discontinuities inside the interval were investigated in [1].

Inverse problems for interior spectral data of the Sturm–Liouville or Dirac operators were studied in [11–13,15]. The paper [12] studied an inverse problem of the regular Dirac operator by using interior spectral data: to reconstruct the potential functions $p(x)$ and $q(x)$ from some known eigenvalues and some information on eigenfunctions at some internal point. In [13] the authors gave a uniqueness theorem for the Dirac operator with discontinuous conditions depending on spectral data of the following kind: to uniquely determine the potential function $q(x)$ by one spectrum and some information on the eigenfunctions at the mid-point of the interval $(0, 1)$.

In the present paper, we study the inverse problem of reconstructing the Dirac operator with discontinuous conditions from spectral data of the following kind: parts of two spectra and some information on the eigenfunctions at some interior point $b \in (1/2, 1)$. The technique we used is similar to those used in [6,12,15].

2. Main results

It is well known that the operator $L(Q(x); a, \alpha, \beta)$ is self-adjoint and has discrete spectrum consisting of simple, real eigenvalues $\lambda_n, n \in \mathbb{Z}$. The sequence $\{\lambda_n\}_{-\infty}^{\infty}$ satisfies the classical asymptotic form [1,8,13]

$$\lambda_n = n\pi - \alpha + \beta + O\left(\frac{1}{n}\right), \quad |n| \to \infty. \tag{2.1}$$

We denote by $y_n(x) = (y_1(x, \lambda_n), y_2(x, \lambda_n))^T$ the eigenfunction corresponding to the eigenvalue λ_n. Together with $L(Q(x); a, \alpha, \beta)$, we consider another problem $\tilde{L}(\tilde{Q}(x); a, \alpha, \beta)$ of the same form but with a different coefficient $\tilde{Q}(x)$ such that

$$\tilde{Q}(x) = \begin{pmatrix} \tilde{p}(x) & \tilde{q}(x) \\ \tilde{q}(x) & -\tilde{p}(x) \end{pmatrix}.$$

It is assumed in what follows that if a certain symbol δ denotes an object related to L, then $\tilde{\delta}$ will denote an analogous object related to $\tilde{L}$.

In the case $b \neq 1/2$, the uniqueness of $Q(x)$ can be obtained from a part of the second spectrum. We denote by μ_n the eigenvalues of the problem $L(Q(x); a, \alpha, \beta_1)$, $\beta_1 \neq \beta, 0 \le \beta_1 < \pi$.

Let $l(n), r(n)$ be sequences of integers with the properties

$$l(n) = \frac{n}{\sigma_1}(1 + \epsilon_{1,n}), \quad 0 < \sigma_1 \le 1, \quad \epsilon_{1,n} \to 0, \tag{2.2}$$

$$r(n) = \frac{n}{\sigma_2}(1 + \epsilon_{2,n}), \quad 0 < \sigma_2 \le 1, \quad \epsilon_{2,n} \to 0. \tag{2.3}$$

Now we state the main result of this work.

Theorem 2.1. *Let $l(n), r(n)$ and $b \in (1/2, 1)$ be such that*

$$\sigma_1 > 2b - 1, \quad \sigma_2 > 2 - 2b. \tag{2.4}$$

If for any $n \in \mathbb{Z}$,

$$\lambda_n = \tilde{\lambda}_n, \quad \mu_{l(n)} = \tilde{\mu}_{l(n)}, \quad \frac{y_1(b, \lambda_{r(n)})}{y_2(b, \lambda_{r(n)})} = \frac{\tilde{y}_1(b, \lambda_{r(n)})}{\tilde{y}_2(b, \lambda_{r(n)})}. \tag{2.5}$$

Then $p(x) = \tilde{p}(x),\ q(x) = \tilde{q}(x)$ for almost all $x \in [0, 1]$.

Remark 2.2. For the problem L without discontinuous conditions, i.e., in the case where $a = 1$ for the conditions (1.3), Theorem 2.1 was already proved in [12].

3. Proof

We shall first mention some results which will be needed later.

Let us denote by $y(x, \lambda) = (y_1(x, \lambda), y_2(x, \lambda))^T$ the solution of Eq. (1.1) satisfy the initial conditions

$$y(0, \lambda) = \begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix}$$

and the jump conditions (1.3). It is shown in [1] that the solution $y(x, \lambda)$ has a representation as follows:

$$y(x, \lambda) = y_0(x, \lambda) + \int_0^x K(x, t) \begin{pmatrix} \sin(\lambda t + \alpha) \\ -\cos(\lambda t + \alpha) \end{pmatrix} dt,$$