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A spectral relaxation method for linear and non-linear stratification effects on mixed convection in a porous medium

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ABSTRACT

In this composition, we use a new spectral relaxation method (SRM) to investigate the effects of linear and non-linear stratification on mixed convective transport along a vertical surface embedded in a porous medium and it is viewed for the first time in both aiding and opposing buoyancy cases. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation and then the resulting differential equations are solved numerically using SRM. A comparison is also made about the accuracy of SRM results in relation to the results obtained using the shooting method. We show that the proposed technique is an efficient numerical algorithm with assured convergence that serves as an alternative to common numerical methods for solving nonlinear boundary value problems. A parametric study of the physical parameters involved in the problem is conducted and a representative set of numerical results is illustrated, with accent on the comparison between linear and non-linear stratification. It is significant to notice that the separation of flow is found to be more in the absence of stratification whereas it is less in the presence of stratification. Finally, thermal and solutal stratifications significantly affect the heat and mass transfer rates, besides delay the boundary layer separation.

1. Introduction

The analysis of mixed convection boundary layer flow on a vertical surface embedded in porous media has received considerable theoretical and realistic contribution. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and thus along. Various authors have examined the problem of mixed convection about different surface geometries and various models were offered to explain mathematical and physical aspects related with the boundary layer flow and convective heat transport in porous media. Among these, the Darcy law gained a good deal of tending. Boundary layer assumptions were successfully employed to these theoretical accounts and much work has been performed on them for different body geometries in the last three decades. A detailed literature concerning these procedures can be establish in recent volumes by Vafai [1], Pop and Ingham [2] and Nield and Bejan [3]. In recent years, several investigators have studied the heat and mass transport problems. Aldoss et al. [4] have considered the magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium. Bourhan [5] analyzed the effects of magnetic and buoyancy on melting

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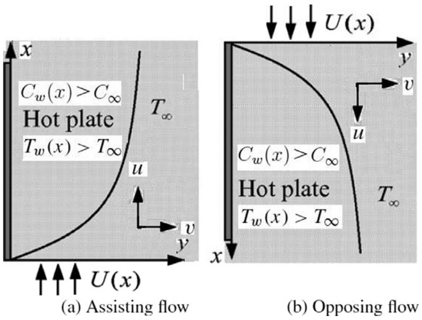

Fig. 1. Physical model and coordinate system.

from a vertical plate embedded in a saturated porous medium. The unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux was studied by El-Kabeira et al. [6].

Since the combined free and forced convection flows arise in a stratified environment in various industrial and technology problems. The input of thermal energy in enclosed fluid regions, due to the emission of hot fluid or heat removal from heated bodies, frequently contributes to the propagation of a stable thermal stratification. Stratification of fluid goes up due to temperature fluctuations, concentration differences or the bearing of dissimilar fluids. It is important to examine the temperature stratification and concentration differences of hydrogen and oxygen in lakes and ponds as they may directly affect the growth rate of all cultured species. Also, the analysis of thermal stratification is important for solar engineering because higher energy efficiency can be achieved with better stratification. It has been shown by scientists that thermal stratification in energy storage may considerably increase system performance. Prandtl [8] first shown that the outcome of the stratification on the mean field is the constitution of a neighborhood with a temperature deficit (i.e. a negative temperature) and flow reversal in the outer portion of the boundary layer of an infinite wall and later on by Jaluria and Himasekhar [9] for semi-infinite walls. Further, the comprehensive survey given by Gebhart et al. [7] have shown that stratification enhances the local heat transfer rate and reduces the velocity and buoyancy levels. In the recent literature, the researchers reported that the temperature and concentration became negative in the boundary layer depending on the relative intensity of the thermal and solutal stratifications (e.g., readers can see the articles given by Rathish Kumar and Shalini [10], Lakshmi Narayana and Murthy [11], Murthy et al. [12] and Srinivasacharya and RamReddy ([16,17])). The mixed convection boundary layer flow through a stable stratified porous medium bounded by a vertical surface is investigated by Ishak et al. [18].

In order to explore theoretical and experimental insight, Jaluria and Gebhart [13] analyzed the stability of the flow adjacent to a vertical plate dissipating a uniform heat flux into a stratified medium. In this study, authors have reported that a theoretical similarity solution exists when the ambient stratification varies like $x^{1/5}$, where x is the downstream coordinate. Unlike the case of linear stratification, the flow reversal and temperature deficit in this case [Gebhart [7]], where the variation of the ambient temperature is relatively weak, are extremely small. But there is no literature which focused on the liner and non-linear stratification.

A novel iteration scheme called the spectral relaxation method (SRM) (see Motsa and Makukula [19], Motsa [20]) is an iterative algorithm for the solution of nonlinear boundary layer problems which are characterized by flow properties that decay exponentially to constant levels far from the boundary surface. The main advantages of the method are the decoupling of the governing nonlinear systems into a sequence of smaller sub-systems which are then discretized using spectral collocation methods. The method is very efficient in solving boundary layer equations of the type under investigation in this study. The current results were validated by comparison with the results obtained using the shooting method. From the literature survey, it appears that the problem of mixed convection from vertical surface in a porous medium in the presence of linear and non-linear stratification under the different wall properties has not been investigated so far and is a fundamental problem of considerable interest. Therefore, this study aims to analyze the effects of linear and non-linear stratification on mixed convection along a vertical surface in a doubly stratified fluid embedded in a porous medium.

2. Mathematical formulation

Consider the two dimensional, laminar and steady mixed convection flow over a heated semi-infinite vertical flat plate in a doubly stratified fluid saturated porous medium. The geometry and the coordinate system are schematically shown in Fig. (1). The x-axis is taken along the plate and y-axis normal to it. The plate is maintained at variable temperature and concentration $T_w(x) = T_{\infty,0} + ax^m$ and $C_w(x) = C_{\infty,0} + bx^m$. The external velocity $U(x)$, the surface temperature $T_\infty(x)$ and the surface concentration $C_\infty(x)$ are assumed to vary as x^m, where x is measured from the leading edge of the vertical surface and m is a constant. The values $T_w(x)$ and $C_w(x)$ are assumed to be greater than the ambient temperature $T_\infty(x)$ and concentration $C_\infty(x)$ respectively at any arbitrary reference point in the medium (inside the boundary layer).