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Finite-time stochastic input-to-state stability of switched stochastic nonlinear systems

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ABSTRACT

In this paper, the problems of finite-time globally asymptotical stability in probability (FGSP) and finite-time stochastic input-to-state stability (FSISS) for switched stochastic nonlinear (SSNL) systems are investigated. To solve these problems elegantly, some new definitions on FGSP and FSISS are presented in the form of generalized $\mathcal{KL}$ ($\mathcal{GKL}$) function, and some lemmas about $\mathcal{GKL}$ functions and their properties are proved. Based on that, some sufficient conditions are provided firstly for nonswitched stochastic nonlinear (nSSNL) systems, which will make the corresponding study on SSNL systems easier. Then, overcoming the difficulties coming with the appearance of switching, some sufficient conditions on FGSP and FSISS are given for SSNL systems. Moreover, based on the concept of average dwell-time, a sufficient condition for FSISS of SSNL systems is also provided. Finally, some simulation examples are given to demonstrate the effectiveness of our results.

1. Introduction

Since the performance of a real control system is affected more or less by uncertainties such as unmodelled dynamics, parameter perturbations, exogenous disturbances, measurement errors etc., the research on robustness of control systems do always have a vital status in the development of control theory and technology. Aiming at robustness analysis of nonlinear control systems, a new method from the point of view of input-to-state stability (ISS), input-to-output stability (IOS) and integral input-to-state stability (iISS) are developed and a series of fundamental results centralizing on the theory of ISS-, IOS-Lyapunov functions are obtained by many scholars, such as Sontag, Wang and Lin, etc. [1], [21], [13], [14], [23], [24], [25], [26], [16], [12]. ISS focuses on the design of smooth controllers to tackle stabilization of various classes of nonlinear systems or their robust and adaptive control in the presence of various uncertainties arising from control engineering applications.

In another active research area, non-smooth (including discontinuous and continuous but not Lipschitz continuous) control approaches have drawn increasing attention in nonlinear control system design. One of the main benefits of the non-smooth finite-time control strategy is that it can force a control system to reach a desirable target in finite time. This approach was first studied in the literature of optimal control. In recent years, finite-time ISS and its applications to finite-time controller design have been considered in many literatures [7], [8], [9], [27]. On the other hand, there are many concerns on stochastic systems for

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the study on stability, controller design, filtering, etc. ([15], [17], [18], [29], etc). But, for the finite-time ISS of stochastic systems, it has not been studied.

From the definition of finite-time input-to-state stability (FISS), it can be found that, if the input $u = 0$, an FISS system is necessarily finite-time globally asymptotically stable (GAS). So, the study of finite-time globally asymptotical stability (GAS) is very helpful to the study of FISS. In [2], [3] and [28], the definition of finite-time globally asymptotical stability in probability (FGSP) was provided and some criteria have been given. But, in [2], [3] and [28], the definition of FGSP was defined in the form of stability in probability plus attractivity in probability. To study the FSISS of stochastic systems, a definition of FGSP in the form of $\mathcal{GKL}$ function ($|x| \le \beta(|x_0|, t)$, where x is the system state and x_0 is the initial value and β is a $\mathcal{GKL}$ function) is needed. The definition of this form is much more elegant and easier to work with.

In this paper, the FGSP and FSISS will be considered for nSSNL and SSNL systems, and the definitions of FGSP and FSISS are both in the form of $\mathcal{GKL}$ function. Firstly, some lemmas are provided to make the proof of our main results easier. Then, for nSSNL systems, the criteria on FGSP and FSISS are provided. To study the FSISS of SSNL systems, the pathwise uniqueness of SSNL systems is considered, and then some criteria on FGSP and FSISS are provided. Moreover, based on the average dwell-time method, a sufficient condition for FSISS of SSNL systems is given. To illustrate the effectiveness of our main results, some simulation examples will be given at the last.

The remainder of this paper is organized as following: Section 2 provides some notations and introduces the definitions of FGSP, FSISS and FSISS-Lyapunov function. Section 3 investigates the FGSP and FSISS property of nSSNL systems. In Section 4, after the pathwise uniqueness of SSNL systems be considered, some criteria on FGSP and FSISS are provided. In Section 5, based on the concept of average dwell time, a sufficient condition for FSISS of SSNL systems is provided. In Section 6, some simulation examples are provided to illustrate the results. Section 7 includes some concluding remarks.

2. Notations and preliminary results

Throughout this paper, $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, n-dimensional real space and $n \times m$ dimensional real matrix space. For vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm $|x| = (\sum_{i=1}^n x_i^2)^{1/2}$. All the vectors are column vectors unless otherwise specified. The transpose of vectors and matrices are denoted by superscript T. $\mathcal{C}([a, b]; \mathbb{R}^n)$ denotes continuous $\mathbb{R}^n$-valued function space defined on $[a, b]$; $\mathcal{C}^i$ denotes all the ith continuous differential functions; $\mathcal{C}^{i,k}$ denotes all the functions with ith continuously differentiable first component and kth continuously differentiable second component. $E(x)$ denotes the expectation of stochastic variable x. The composition of two functions $\varphi: A \to B$ and $\psi: B \to C$ is denoted by $\psi\circ\varphi: A \to C$.

A function $\varphi(u)$ is said to belong to the class $\mathcal{K}$ if $\varphi \in \mathcal{C}(\mathbb{R}_+, \mathbb{R}_+)$, $\varphi(0) = 0$ and $\varphi(u)$ is strictly increasing in u. $\mathcal{K}_\infty$ is the subset of $\mathcal{K}$ functions that are unbounded. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$, if $\beta(\cdot, t)$ is of class $\mathcal{K}$ in the first argument for each fixed $t \ge 0$ and $\beta(s, t)$ decreases to 0 as $t \to +\infty$ for each fixed $s \ge 0$.

A function $h : \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to the class generalized $\mathcal{K}$ ($\mathcal{GK}$) if it is continuous with $h(0) = 0$, and satisfies

$$\begin{cases} h(r_1) > h(r_2), & \text{if } h(r_1) \ne 0; \\ h(r_1) = h(r_2) = 0, & \text{if } h(r_1) = 0, \end{cases} \quad \forall r_1 > r_2. \tag{1}$$

Note that a class $\mathcal{GK}$ function is a (conventional) class $\mathcal{K}$ function, which is defined as a continuous and strictly increasing function with $h(0) = 0$ because a strictly increasing function satisfies (1). A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class generalized $\mathcal{KL}$ function ($\mathcal{GKL}$ function) if, for each fixed $t \ge 0$, the function $\beta(s, t)$ is a generalized $\mathcal{K}$-function, and for each fixed $s \ge 0$ it decreases to zero as $t \to T$ for some $T \le \infty$.

Consider the following n-dimensional stochastic nonlinear (SNL) system

$$dx = f(t, x, u)dt + g(t, x, u)dw,\ t \ge t_0, \tag{2}$$

where $x \in \mathbb{R}^n$ and $u \in \mathcal{L}_\infty^m$ are system state and input, respectively; $\mathcal{L}_\infty^m$ denotes the set of all the measurable and locally essentially bounded input $u \in \mathbb{R}^m$ on $[t_0, \infty)$ with norm

$$\|u\| = \sup_{t\ge t_0} \inf_{\mathcal{A}\subset\Omega, P(\mathcal{A})=0} \sup\{|u(t, \omega))| : \omega \in \Omega\backslash\mathcal{A}\}. \tag{3}$$

$w(t)$ is an r-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge t_0}, P)$, with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\ge t_0}$ being a filtration and P being a probability measure. $f : [t_0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $g : [t_0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n\times r}$ are continuous and satisfies $f(\cdot, 0, 0) \equiv 0, g(\cdot, 0, 0) \equiv 0$. Moreover, system (2) is assumed to has a pathwise unique strong solution[19], denoted by $x(t, t_0, x_0)$, $t_0 \le t < +\infty$, for any given $x_0 \in \mathbb{R}^n$.

For convenience, we denote system (2) with input $u = 0$ as follows

$$dx = f(t, x)dt + g(t, x)dw,\ t \ge t_0, \tag{4}$$

and introduce some corresponding definitions on FGSP and FSISS.

Definition 2.1. (Stochastic settling time function). For system (4), define $T_0(t_0, x_0, w) = \inf\{T \ge 0 : x(t, t_0, x_0) = 0,\ \forall t \ge t_0 + T\}$, which is called the stochastic settling time function. Especially, $T_0(t_0, x_0, w) =: +\infty$ if $x(t, t_0, x_0) \ne 0$, $\forall t \ge t_0$.