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Thermoelastic analysis for rotating circular HSLA steel plates with variable thickness

Hong-Liang Dai [a,b,*], Ting Dai [a,b], Xiang Yan [a,b,c]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, 410082, China
[b] Department of Engineering Mechanics, College of Mechanical & Vehicle Engineering, Hunan University, Changsha, 410082, China
[c] Zhuzhou Times New Material Technology CO.LTD, Zhuzhou, 412000, China

A R T I C L E I N F O

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A B S T R A C T

Thermoelastic analysis for a rotating circular HSLA (high strength low alloy) steel plate with variable thickness, which is placed in a temperature field, and subjected to a mechanical load is presented. Based on the von Karman equation and classical thin plate theory, the nonlinear governing equations for the mid-plane displacements of the rotating circular HSLA steel plate are obtained by using the Hamilton variation principle. The displacements and rotation angle are discretized in space and time domains by utilizing the finite difference method and Newmark method, and the whole problem is solved by the iterative method. Numerical results show that the geometric shape, angular speed, mechanical load and temperature field all have great influence on the thermoelastic behavior of the rotating circular HSLA steel plate.

1. Introduction

In recent years, as the development of automobile industry, energy crisis and environmental problem are aggravating, hence, more attentions are attached to lightweight, safety, environment protection and fuel economy in automobile industry. In order to meet the requirements of lightweight, saving fuel, reducing exhaust gas emission and improving vehicle crash safety, the use of HSLA steel plates in automobile industry becomes more and more popular. In real applications, various detail structures, such as axial rotation and variable thickness, of the HSLA steel plates can be seen for special uses. Meanwhile, this kind of plate is always under changing temperature environment, especially when it is being heat-treated or laser processed. Hence, it will be of great importance to analyze the thermoelastic behaviours of rotating circular HSLA steel plates with variable thickness, and this work can be a useful reference for the designs of both the HSLA steel plate structures and the heat treatment or laser processes of them.

In fact, researches on HSLA steel structures are not rare. Rusinek et al. [1] analyzed thermoviscoplastic behavior and the micro-mechanism of plastic deformation of HSLA steels in a wide range of strain rates and temperatures. Yan et al. [2] presented change of tensile behavior of a HSLA steel with tempering temperature. Wang et al. [3] investigated microstructures and critical phase-transformation temperature of boron–nickel added Nb-treated HSLA H-beams cooled at different cooling rates. Based on the thermoviscoelastic theory, classical plate theory and dynamics principle, Dai et al. [4,5] investigated thermoviscoelastic behavior and dynamic responses in a circular HSLA steel plate. Lee et al. [6] carried out a numerical simulation for spring-back and spring-go behaviors in bending of thick plates of high-strength steel at elevated temperature. About the thermodynamic stresses and

* Corresponding author. Tel.: +8673188664011; fax: +8673188711911.
E-mail address: hldai520@sina.com (H.-L. Dai).

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deformations of a HSLA steel plate, Jiang et al. [7–9] did a lot of investigations. From the literatures, we find that the shapes of researched HSLA steel plates are square, rectangle, strip, circular or annular. However, circular plate with variable thickness, which is one of the most common seen structures in industrial applications, is rarely found to be theoretical investigated.

Rotating circular plate is a kind of structure commonly seen in practical applications, but always, the geometric shape of it is designed for special uses, such as variable thickness. Studies on dynamical behaviors of rotating circular plates with variable thicknesses are abundant. Utilizing the finite element method, Reddy and Huang [10] presented the large deflection bending of annular plates with variable thickness. Applying the Mendelson's method of successive elastic solution, Loghman et al. [11] investigated the time-dependent creep stress redistribution analysis of rotating disk made of Al-SiC composite. Based on the J2 deformation theory, Hu [12] analyzed large deformation of the axisymmetric rotating disk. Aleksandrova [13] obtained the continuous stress-displacement solution to thin rotating solid disk problem for elastic-plastic material. Calderale et al. [14] put forward a theoretical thermoelastic analysis of the Stodola's hyperbolic disk, axisymmetric and symmetric with respect to the mid-plane, and subjected to thermal load. By using Seth's transition theory, Thakur [15] derived the stresses for the elastic-plastic transition and fully plastic state for a thin rotating disc with rigid shaft having variable thickness, after that, Thakur et al. [16] investigated the same structure with density variation parameter under steady-state temperature. With the combination of Runge–Kutta's and finite element methods, Hassani et al. [17] carried out both theoretical and numerical analyses for rotating disks with non-uniform thickness and material properties subjected to thermo-mechanical loadings. Based on the linear, small strain, three-dimensional elasticity theory, the free vibration of annular thick plates with linearly varying thickness is studied by Zhou and Lo [18]. Aiming at finding an optimal disk profiles for minimum weight design, Jafari et al. [19] analyzed the rotating disk using the Karush–Kuhn–Tucker method, simulated annealing, and particle swarm optimization. Using a three-dimensional numerical method, Zhang et al. [20] investigated the flow and heat transfer characteristics over a rotating disk with bottom wall subjected to a uniform heat flux. Taking into account paradoxes of elastic–perfectly plastic material model, Aleksandrova [21] considered the problem of solid rotating disk from the perspective of delivering continuous stress–strain analytical solution for engineering applications. However, as far as we know, specific to the material of HSLA steel, thermoelastic behaviors of a rotating circular plate with variable thickness have not been theoretically studied.

In this paper, thermoelastic analysis of a rotating circular HSLA steel plate with variable thickness, placed in a temperature field, subjected to a mechanical load is presented. Based on the von Karman equation and classical thin plate theory, the non-linear governing equations on displacement of the mid-plane for the rotating circular HSLA steel plate are obtained by using the Kirchhoff hypothesis, and the whole problem is solved by applying the finite difference method, Newmark method and iterative method.

2. Mechanical model and basic equations of the problem

Consider a rotating circular HSLA steel plate with variable thickness, which is placed in the temperature field $\Delta T(r)$, and subjected to an axisymmetric transverse mechanical load $p(t)$. The inner and outer radius of the circular HSLA steel plate is r_1 and r_2, respectively. ρ is used to represent the mass density and ω denotes the rotating angular speed. The thickness of the circular HSLA steel plate is supposed to obey the following expression

$$H(r) = H_0\left[1 - a\left(\frac{r}{r_2}\right)^n\right] \tag{1}$$

where a $(0 \le a < 1)$ and n are geometric parameters for the variable thickness of the circular HSLA steel plate, and H_0 is the thickness at the center of the circular plate. Given this, when $a = 0$, the thickness of the circular plate is uniform. When $a \neq 0$, there will be three kinds of thicknesses as shown in Fig. 1. That is, if $n = 1$ the thickness is linear decreasing from the center to the boundary as in Fig. 1a, if $n > 1$ the outline of the circular HSLA steel plate is concave as in Fig. 1b, and if $n < 1$ it is convex as in Fig. 1c. For convenient description of the whole problem, a cylindrical coordinate system (r, θ, z) is established, as shown in Fig. 2.

Based on the general Hooke's law, and taking into account the temperature environment, the stress-strain relations for the circular HSLA steel plate are

$$\begin{bmatrix}\sigma_r\\ \sigma_\theta\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu\\ \nu & 1\end{bmatrix}\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\end{bmatrix} - \frac{E\alpha}{1-\nu}\begin{bmatrix}\Delta T\\ \Delta T\end{bmatrix} \tag{2a}$$

$$\sigma_{rz} = \frac{E}{2(1+\nu)}\varepsilon_{rz} \tag{2b}$$

where σ_i and ε_i $(i = r, \theta, rz, z, r\theta, \theta z)$ separately denote the stress and strain components of the circular HSLA steel plate, and E, ν and α is the Young's modulus, Poisson's ratio and thermal expansion coefficient, respectively.

Considering the nonzero transverse shear strain, and based on the first-order shear deformation plate theory, the radial displacement $u_r(r, z, t)$ and transverse displacement $w_z(r, z, t)$ of the circular HSLA steel plate can be expressed as

$$u_r(r, z, t) = u(r, t) + z\phi(r, t) \tag{3a}$$

$$w_z(r, z, t) = w(r, t) \tag{3b}$$