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A reaction-diffusion equations modelling the effect of fluctuating water levels on prey-predator interactions

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ABSTRACT

Prey-predator interactions are influenced by many ecological factors. In this paper, a prey-predator model with spatial diffusion under homogeneous Neumann boundary conditions is investigated to study the impact of water level on persistence of two fish populations living in an artificial lake. We obtain some important qualitative properties, including the existence of the global positive solution, the dissipation and permanence of this system. The existence of a unique globally stable periodic solution is also presented. Some numerical examples are presented to verify our results.

1. Introduction

In order to precisely describe the real impact of water level fluctuations on the interactions between the species in rivers, lakes and reservoirs, Chiboub et al. [5] developed a new model in which they incorporated the variation of water level in a system of differential equations. In the proposed model, they used a Prey-Predator model describing the interaction between two fish species in Pareloup lake. The Pareloup lake is a reservoir where water is taken partly from Vioulou, a river located at 30 km in the southeast of Rodez. It is among the largest artificial lake in France, allowing to store water during seasons of high electricity demand (see Fig. 1). The management of this lake is of considerable ecological importance. Significant variations of the water level of the lake can have a strong impact on the persistence of some species [7,22]. In fact, the increase of water volume hinders the capture of the prey by the predator. The same reasoning is applied when there is a decrease in the volume of water, favoring the capture of the prey by the predator; the authors used the population densities of the Pike species (Gardon in French) as prey and the Roach species (Brochet in French) as the predator, the pike and roach are the most important species in this lake. They studied the dynamic behaviors of the following predator-prey system:

$$\begin{cases} \dfrac{dG}{dt}(t) = G(t)(\gamma_G - m_G G(t)) - \min\left(r(t)\dfrac{G(t)}{B(t)+D}, \gamma_B\right)B(t) \\ \dfrac{dB}{dt}(t) = \tau_B \min\left(r(t)\dfrac{G(t)}{B(t)+D}, \gamma_B\right)B(t) - m_B B(t) \end{cases} \tag{1}$$

where $G(t)$ and $B(t)$ are respectively the densities of the prey and predator at time t and $r(t)$ is the accessibility function for the prey. When a predator attacks a prey, it has access to a certain quantity of food depending on the water level. When water level is low the predator is more in contact with the prey. It is assumed that the function $r(t)$ is annual periodic and continuous, that

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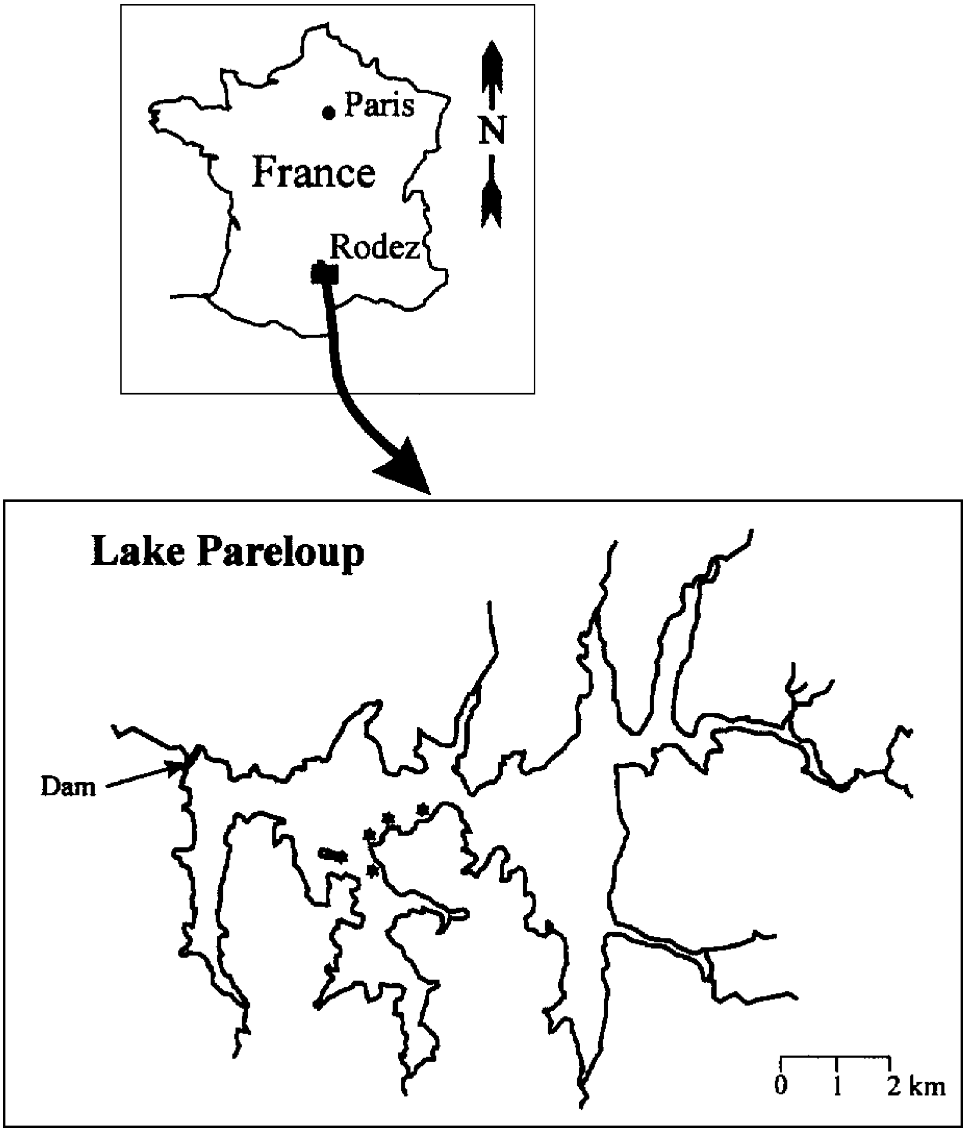

Fig. 1. Map of France showing location of lake Pareloup and a representation of the reservoir.

is, r is 1-periodic, the minimum value r_1 is reached in spring and the maximum value r_2 is attained during autumn. The predator needs a quantity γ_B for his food, but he has access to a quantity

$$\frac{r(t)G}{B+D},$$

here D measures the other causes of mortality outside the metabolism and predation. If

$$\frac{r(t)G}{B+D} \geq \gamma_B,$$

then the predator will be satisfied with the quantity γ_B for his food. Otherwise, i.e., if

$$\frac{r(t)G}{B+D} \leq \gamma_B,$$

the predator will content himself with

$$\frac{r(t)G}{B+D}.$$