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Mould-taper asymptotics and air gap formation in continuous casting

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ABSTRACT

We develop a coupled thermomechanical model, that includes mould taper, for the formation of air gaps in the vertical continuous casting of round billets. The system is very sensitive to the small width of the air gap. Mould tapers are used to mitigate the contraction of the solidified shell during cooling. We apply numerical and perturbation methods to show that a small mould taper significantly reduces the insulating effect of the air gap. The analysis is presented in a more transparent and less computationally expensive way than earlier, fully numerical models. We also consider a theoretical ideal taper, which eliminates the air gap altogether. The air gap is found to be quite robust; increasing the size of the taper does not constitute an equal reduction in the air gap size. Sample computations are carried out using parameters for the continuous casting of a pure metal (copper), although the framework can easily be extended to the continuous casting of alloys.

1. Introduction

The formation of an air gap between the cooled mould and solidified shell in the industrial continuous casting of metals (Fig. 1) and metal alloys has long been recognized as having an adverse effect on process efficiency [1–9]. The gap arises because the solidified metal shell contracts as it cools; one way to mitigate this effect is to taper the mould.

In view of the complex interaction between cooling shrinkage, air gap formation and mould tapering, as well as possible thermal distortion of the mould, mathematical models of varying degrees of complexity have been derived to describe this situation [10–12]. Without exception, these models are solely numerical; however, whilst able to capture the thermomechanical interaction of air gap formation and evolution, these models are computationally expensive and unwieldy, and give scant qualitative understanding of the air gap's dependence on different operating parameters.

Recently, an alternative approach has been suggested [13–17], for the problem without mould taper, that uses asymptotic methods to reduce the complexity of the problem, although without sacrificing any physical features. This is possible because the thickness of the air gap is much smaller than all other length scales in the problem. Similarly, the extent to which a mould is tapered – around 2%/m for the case of the continuous casting of steel [2] – also suggests that the situation should be amenable to an asymptotic treatment. As has been highlighted previously [13–17], there are two obvious benefits in adopting the asymptotic

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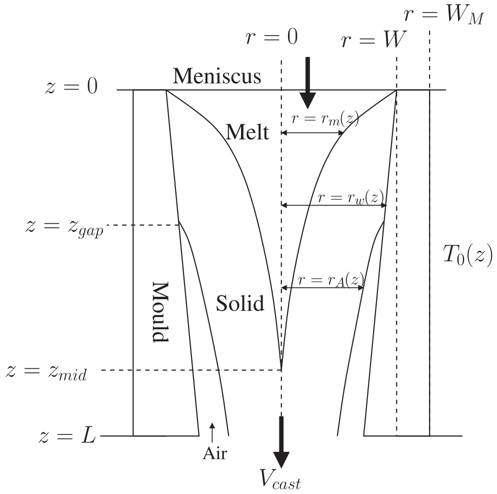

Fig. 1. A 2D schematic of the continuous casting process with tapered mould walls.

route: for initial design purposes, to gain insight prior to more general and expensive simulations, and for use as a benchmark in testing more general formulations.

In particular, we extend upon the axisymmetric model presented by Vynnycky [15]. Whilst it may seem that adding a mould taper is a simple geometric perturbation, due to the extreme sensitivity of the heat transfer to the width of the air gap, a small taper can have a large effect on the solidification rate.

There are two main ways by which the taper can affect the air gap: by delaying the formation of the air gap and simply thinning the air gap by occupying the air space. In our analysis, we will seek to address both possibilities. We also consider the concept of the ideal taper, which is the minimal taper required to completely eliminate the air gap. The size and profile of this taper is important to taper design.

The layout of the paper is as follows. In Section 2, we formulate the appropriate thermomechanical problem for the continuous casting of round billets in which the generalized plane-strain approximation holds. In Section 3, we nondimensionalise the governing equations and perform a self-consistent asymptotic reduction near the formation of the air gap to explore the effectiveness of a mould taper to delay the formation. In Section 4, a finite-difference method is used to solve the governing equations, which constitute a moving-boundary problem for the temperature, along with a boundary condition in integro-differential form that describes the evolution of the air gap and how effective the taper is at reducing its width. Finally, conclusions are drawn in Section 5.

2. Mathematical formulation

We consider a steady-state vertical caster with cylindrical geometry, as shown in Fig. 1. A pure molten metal enters at $z = 0$ at its melting temperature T_{melt}, and immediately begins solidifying at the mould surface, such that $r_m(0) = W$, where $r_m(z)$ is the solidification front as a function of the vertical depth, z, and W is the radius of the caster. The tapering of the inner mould wall is given by $r_w(z)$, which is defined, without loss of generality, such that $r_w(0) = W$. An air gap forms at $z = z_{\text{gap}}$, and complete solidification occurs at $z = z_{\text{mid}}$. For $0 \le z \le z_{\text{gap}}$, the solid cast is confined to $r_m(z) \le r \le r_w(z)$ whereas for $z_{\text{gap}} < z \le z_{\text{mid}}$, the solid occupies $r_m(z) \le r \le r_A(z)$, where $r_A(z)$ is the interface between the solid cast and the air gap. After complete solidification, i.e., for $z > z_{\text{mid}}$, the solid occupies $0 \le r \le r_A(z)$. The outer surface of the mould wall is held at the temperature, $T_o(z)$, which can, in practice, be measured experimentally by thermocouples located within the mould itself.

2.1. Heat transfer

The system is modelled at steady state where we assume that the process has been running for enough time for transient solutions to have disappeared. This simplification allows the use of z as a time-like variable, related to t by the casting speed, V_{cast}. Thus, the heat-transfer is modelled as follows. For $0 < z < z_{\text{gap}}$ and $0 < r < r_m(z)$, and then $z > z_{\text{gap}}$ and $r_A(z) < z < r_m(z)$, the radially symmetric heat equation is given by

$$\rho c_{ps} V_{\text{cast}} \frac{\partial T_s}{\partial z} = \frac{k_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right), \tag{2.1}$$

where T_s is the temperature of the solid, k_s is the thermal conductivity of the solid metal, c_{ps} is its specific heat capacity and ρ its density. Here we use the fact that casting geometries are typically slender, justifying the assumption that $\partial^2/\partial z^2 \ll \partial^2/\partial r^2$.