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Numerical investigation on influence of real gas properties on nonlinear behavior of labyrinth seal-rotor system

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ABSTRACT

A nonlinear model of fluid-structure interaction between high-pressure methane leakage through interlocking seal and the whirling rotor was proposed. The real gas properties of the methane at the pressure 10^2 bar were considered in the mathematical reduction. Three cases of different pressure ratio 1.7, 2.5 and 3.3 at the constant inlet pressure 250 bar were chosen in the present study. Two models, e.g., ideal gas model and real gas model, were employed to investigate the influence of real gas properties of methane leakage on the rotor dynamics. Distribution of thermal parameters in the seal cavities and seal clearance were determined, e.g., density, temperature, compressibility factor and specific heat capacity. The rotor-seal system was modeled as a Jeffcot rotor subject to shear stress and pressure force associated with the methane gas leakage. Spatio-temporal variation of the methane gas forcing on the rotor surface in the coverage of the seal clearance and the cavity volume was calculated by using the Muzynska model and the perturbation analysis, respectively. The governing equation of rotor dynamics which includes the main contribution from the methane leakage forcing was solved by using the fourth-order Runge–Kutta method, resulting in the orbit of the whirling rotor.

1. Introduction

Methane gas compressors have seen considerable development at ever-increasing discharge pressure on the order of $10^2\sim10^3$ bars. However, the methane gas leakage flow through the seals, which are placed between the rotor and the stator, would have a significant impact on the whirling rotor. With increasing pressure of the methane fluid, the flow-structure interaction between the rotor and the methane leakage may exhibit nonlinear behaviors. Accordingly, a model of accurately predicting nonlinear behaviors associated with the fluid-structure interaction between the high-pressure methane leakage and the whirling rotor is necessary.

Various efforts have been attempted to establish mathematical models in a view to see influence of leakage flow through labyrinth seal on rotordynamics. Through perturbation analysis [1] and single-control-volume bulk flow model, Kostyuk [2] and Iwatsubo [3] presented a linearized force-displacement model to see aerodynamic influence of the leakage air flow on rotor-dynamics. Subsequently, rotordynamic coefficients associated with the leakage air flow through various labyrinth seals were

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Nomenclature

A_n	Unsteady cross-sectional area of the cavity (m^2).
A_0	Steady annular flow area (m^2).
B	Tooth height (m).
C_0	Orifice contraction coefficient.
C_1	Kinetic energy carry-over coefficient.
C_r	Steady radial clearance (m).
Dh_0	Steady hydraulic diameter of the cross-sectional area of the cavity (m).
Dh	Unsteady hydraulic diameter of the cross-sectional area of the cavity (m).
D_e	Damping coefficient of the rotor (N s/m).
D_i	Equivalent damping of methane gas flow at the *i*th seal clearance (N s/m).
D_{br}	Equivalent damping of oil film in the journal bearings (N s/m).
e	Displacement of the rotor from the central position (m).
$F(t)\|_{\mathrm{ca}}$	Total methane gas reaction force in the cavities (N).
$F_x(t)\|_{\mathrm{br}}$	Journal bearing reaction force component in the *X* direction (N).
$F_y(t)\|_{\mathrm{br}}$	Journal bearing reaction force component in the *Y* direction (N).
$F_x(t)\|_{\mathrm{ca}}$	Cavity reaction force component in the *X* direction (N).
$F_y(t)\|_{\mathrm{ca}}$	Cavity reaction force component in the *Y* direction (N).
$F_x(t)\|_{\mathrm{cl}}$	Seal clearance reaction force component in the *X* direction (N).
$F_y(t)\|_{\mathrm{cl}}$	Seal clearance reaction force component in the *Y* direction (N).
$h_{0i\mathrm{seal}}, h_{i\mathrm{seal}}$	Steady and unsteady local enthalpy at the *i*th clearance (J/Kg).
h_0	Stagnant enthalpy (J/Kg).
H	Unsteady interlocking seal radial clearance (m).
$H_1(t, \theta)$	Perturbation clearance (m).
L	Pitch of the cavity (m).
m_0	Leakage flow through the interlocking seal (kg/s).
m_r	Mass of the rotor (kg).
m_{fi}	Equivalent mass of methane gas flow at the *i*th seal clearance (kg).
m_{fbr}	Equivalent mass of oil film in the journal bearings (kg).
N	Tooth number.
n	Pressure ratio of the interlocking seal.
$P_{0i\mathrm{seal}}, P_{i\mathrm{seal}}$	Steady and unsteady pressure at the *i*th seal clearance (Pa).
P_{0i}, P_i	Steady and unsteady pressure in the *i*th cavity (Pa).
$P_{1i}(t, \theta)$	Perturbation pressure in the *i*th cavity (Pa).
$\dot{q}_{0i}, \dot{q}_i$	Steady and unsteady leakage rate per unit length (kg/s m).
$\dot{q}_{1i}(t, \theta)$	Perturbation leakage flow rate per unit length (kg/s m).
R_s	Rotor radius (m).
R	Gas constant (J/kg-K).
T_i	Temperature at the *i*th cavity (K).
$T_{i\mathrm{seal}}$	Temperature at the *i*th orifice (K).
$U_{0i\mathrm{seal}}$	Steady axial velocity at the *i*th seal clearance (m/s).
V_{0i}	Steady circumferential velocities at the *i*th cavity (m/s).
V_i	Unsteady circumferential velocities at the *i*th cavity (m/s).
$V_{1i}(t, \theta)$	Perturbation pressure at the *i*th cavity (m/s).
W	Width of tooth (m).
$(x_{o'}(t), y_{o'}(t))$	*x* and *y* coordinates of the whirling rotor (m).
$Z_i, Z_{i\mathrm{seal}}$	Compressibility factors in the *i*th cavity and at the *i*th orifice.

Greek symbols

α_r	Dimensionless shear stress length at the rotor wall.
α_s	Dimensionless shear stress length at the stator wall.
ε	Eccentricity ratio.
η	Imaginary number.
θ	Azimuthal position.
$\rho_{0i\mathrm{seal}}, \rho_{i\mathrm{seal}}$	Steady and unsteady gas density at the *i*th clearance (kg/m^3).
$\rho_{1i\mathrm{seal}}(\theta, t)$	Perturbation gas density at the *i*th clearance (kg/m^3).
ρ_{0i}, ρ_i	Steady and unsteady gas density at the *i*th cavity (kg/m^3).
$\rho_1(\theta, t)$	Perturbation gas density of the *i*th cavity (kg/m^3).