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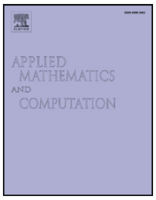

The impact of initial cooperation fraction on the evolutionary fate in a spatial prisoner's dilemma game

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ABSTRACT

Although the initial cooperation fraction and its arrangement obviously impacts the equilibrium in spatial prisoner's dilemma (SPD) games, most of the previous studies relying on a simulation approach have by conformity assumed a value of 0.5, thus implying an equal distribution of cooperators and defectors as the established premise. This study first points out that depending on target applications, i.e., biological or general evolutionary dynamical systems, we must select the initial cooperation fraction and its spatial distribution carefully to avoid a misleading conclusion. On the basis of comprehensive numerical experiments and a deductive discussion, wherein both the rule of strategy update and the underlying network are varied, we present a panoramic view of the effects of initial cooperation settings on evolutionary dynamics. Several non-trivial findings are revealed concerning the different impacts of deterministic or stochastic updating, or of homogeneous or heterogeneous underlying topology on evolutionary dynamics.

1. Introduction

Evolutionary game theory (EGT) has attracted a lot of attention in the last several decades as an interdisciplinary field related to various other fields such as biology, physics, information science, and social science, including economics and politics. EGT postulates that both theoretical and numerical approaches, using simplified templates such as prisoner's dilemma (PD) and the snowdrift game (chicken game), may shed some light on plausible answers to the mysterious puzzle of why human beings, as well as other animal species, successfully evolve autistic cooperation rather than egocentric defection within their societies (e.g., [1]).

Let us limit the following discussion to two-strategy games, especially 2 × 2 (two-player and two-strategy) games, where two agents independently offer either cooperation (C) or defection (D) to play a game. Assuming an infinite and well-mixed population and relying on so-called replicator dynamics, we know that the equilibrium of a game's dynamics can be classified into four classes: PD, chicken, stag hunt (SH) and trivial depending on its payoff matrix: $\begin{bmatrix} R & S \\ T & P \end{bmatrix}$. Only an SH game (with features $D_g \equiv T - R < 0$ and $D_r \equiv P - S > 0$ [2]) has a bi-stable equilibrium state, implying that the dynamics bifurcate into either an all-cooperators state or all-defectors state depending on the initial cooperation fraction. In a nutshell, none of the dynamics belonging to the other three classes: PD, chicken and trivial, show dependence on the initial strategy distribution (say the initial cooperation fraction). All these three games let the dynamics be absorbed by a single equilibrium state of either all-cooperators, all-defectors or internal equilibrium where cooperators and defectors coexist, irrespective of the initial cooperation fraction. This

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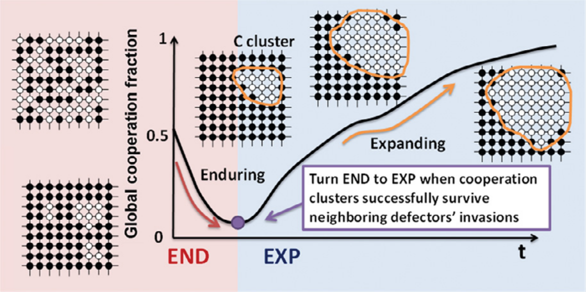

Fig. 1. Schematic view of the evolution of cooperation in the spatial prisoner's dilemma game with the concept of END and EXP, which was reported in [7] and [8]. *Enduring (END) period*: Initial cooperators will be rapidly plundered by defectors, which cause only a few cooperators to survive through forming compact C-clusters. *Expanding (EXP) period*: C-clusters start to expand, since a cooperator on the clusters' border can attract a neighboring defector into the cluster. We presume $P_C^{\text{initial}} = 0.5$ for this visualization. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

fact, however, becomes untrue when any social viscosity mechanism [3] to foster cooperation is additionally presumed in the original game model. In such cases, because we can no longer use an analytical approach based on replicator dynamics, we must rely on a numerical approach, in which we statistically argue asymptotical equilibria based on many independent simulation trials. In these situations, note that the initial cooperation fraction significantly affects the dynamics, which is quite different from the analytic approach case mentioned above. Paraphrasing this, we should point out that the dynamics of general evolutionary games are attributed to both the game structure instinctively stipulated by a payoff-matrix and the initial cooperation fraction. The influence resulting from the initial cooperation fraction is no longer negligible.

Herein, let us concern ourselves with how the setting of the initial cooperation fraction has been treated in view of the applications that are expected from previous simulation studies. In evolutionary biology, it has been accepted as the most important, central principle that a new species that successfully acquires, through mutation, a superior trait in terms of fitness expels the domestic species. Therefore, in biological applications, the so-called fixation probability occupies an important place. First-order fixation probability, ρ_C (ρ_D), is the probability that a mutant's strategy C (D), originated from a single mutant of C (D), can occupy the entire population of size N by replacing resident agents of D (C). We can evaluate the evolutionary stability of cooperation by comparing ρ_C with ρ_D as well as with $1/N$, indicating a probability that a single mutant, defined as evolutionarily neutral, and the mutant's descendants, finally dominate the entire population. For example, if $\rho_D > \frac{1}{N} > \rho_C$, it can be concluded that defection is evolutionarily stable because a defector can easily invade cooperative residents but a cooperator cannot invade defective residents, as the probability is less than that caused by random drift. To numerically evaluate ρ_C (ρ_D), the initial cooperation fraction in a series of simulations is set to $1/N$ ($\frac{N-1}{N}$).

Meanwhile, in reviewing a huge number of previous simulation studies not addressing biological applications but targeting general problems of evolutionary games, we notice that the vast majority of them have assumed the initial cooperation fraction is 0.5, which implies an equal fraction of cooperators and defectors. In this case it would be interesting to observe the dynamics when the problem starts from the parity situation, rather than from an unequal situation, for example, assuming a single defector among many cooperative residents. This standpoint is based on the idea that we are primarily concerned with the dynamics of a two-strategy system, not on the biological population dynamics competed by cooperators and defectors. In a sense, this is going along with the system dynamics concept.

In these cases we can see that the initial cooperation fraction that should be assumed might be different depending upon application, i.e., biological applications or pure system dynamics concepts. However, note that there have been no comprehensive discussions on how an initial cooperation fraction influences its evolutionary path, other than an exception mentioned later. This is because the initial cooperation fraction has been treated as just one of the initial assumptions, which might be shared by researchers as a covert agreement. This seems superficial and it may mislead our discussion to an erroneous conclusion, because we cannot deny the possibility that a game, statistically exhibiting the all-defectors state as long as it is provided an initial cooperation fraction of 0.5, may show a C and D co-existing phase when another initial cooperation fraction, 0.3, for example, is assumed. Shall we assume this particular game defection is dominant? Of course not. More deliberate investigation is required.

The present study concerns the influence of the initial cooperation fraction on the evolutionary trajectory. Our focus is limited to the dynamics of 2 × 2 games played on a network, which indicates the dynamics are affected by the so-called network reciprocity. As Nowak insisted in his milestone paper [3], network reciprocity is one of the five fundamental mechanisms for resolving the dilemma by adding social viscosity. Network reciprocity has continued to receive intensive attention (for comprehensive reviews, refer to [4–6], and for other related works [7–11]) because—although the central assumption of the model, i.e., "playing with neighbors on an underlying network and copying a strategy from them," is simple—it still seems very plausible for explaining why cooperation survives in any real context.

In our recent studies, we have provided a holistic discussion on the substantial mechanism that leads to network reciprocity [12–14]. The key idea is that an evolutionary course transitioning from an initial random state to a final equilibrium state is divided into two temporal periods, and we carefully observe the mechanism in these two periods as shown in Fig. 1. Following