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Numerical solution of the steady-state probability and reliability of a repairable system with three unites

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ABSTRACT

In the present paper, a series-parallel repairable system with three unites was developed under some assumptions. By using the supplementary variables method, probability arguments and limiting transitions, the integro-differential equations governing the behavior of the system were obtained. Since some of the system equations have two hazard functions involved, the numerical simulation methods were used to analyze the reliability of the system. Firstly, combining the boundary conditions and characteristic curve method, the state equations were transformed into a set of integral equations. Secondly, the sequences of approximating functions were constructed for the integral equations which are analogous to the Gauss–Seidel or SOR iterative scheme for solving a system of linear equations. It was showed that the sequences of approximating functions converge pointwise to the solution of integral equations and they have continuously differential solutions. Finally, the numerical solutions of the integral equations, system equations and some reliability indices were presented by building a discretization of approximating sequence.

1. Introduction

Repairable system is not only an important class of systems discussed in reliability theory but also one of main objects studied in reliability mathematics. It consists of some components under the supervision of one or more repairmen. If a component fails at any time, it is immediately sent to the repair facility for repair.

The repairable system with three units is frequently encountered in practice. Many authors have discussed this repairable system in the past two decades. For instance, Song and Deng analyzed the reliability of a three-unit system in a changing environment in [1]. Li et al. studied a repairable system with three units and two different repair facilities and derived the explicit expressions of the state probabilities of the system and the steady-state reliability characteristics of the system in [2]. Kovalenko investigated a three-component system consisting of one master control element and two slave elements with priority serving by a single repair facility and obtained the readiness factor and the average up-time in [3]. Hu et al. discussed a three-unit system with n failure modes and priority and obtained the explicit expressions of the steady-state probabilities of the system in [4]. Moreover, Hu et al. studied the steady-state availability and the mean up-time of a series-parallel repairable system consisting of one master control unit, two slave units and a single repairman who operates single vacation in [5]. Recently, Chen studied the three-unit repairable system with preemptive priority to part 1 and single vacation of a repairman in [6].

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In the researches above, the failure rates or repair rates were assumed constant. Whereas, from the viewpoint of practice, the constant failure rate assumption for components has been and is repeatedly challenged by knowledgeable reliability practitioners. Therefore, there are other studies which handled the problem of time-varying failure rates [7]–[11]. In most cases, however, time-varying failure and repair rates in system reliability and availability analysis is the most appropriate for real systems. But it is very difficult to obtain the analytic solution and, in most cases, the closed-form solution for system reliability and availability does not exist, so that numerical or simulation methods must be used.

In the present paper, we investigate a three-dissimilar-unit model with one repair facility to show how to obtain the undetermined functions when the supplementary variables method (SVM) is used. This model is a difficult one to analyze since there are some random variables having a general distribution. For the model considered here, some of the system equations have two hazard functions involved and the process of using the SVM then involves some functions which need to be determined. There is no general method to deal with this problem and in many cases it is almost impossible to derive explicit results from the system equations (see [12,13]). Motivated by the works of [8]–[11], we construct several numerical approximations to the solution of the model and use these numerical algorithms to examine its behavior. Furthermore, we obtain explicit expressions for some main availability indices of the system and the system's reliability.

The content of this paper is organized as follows. In Section 2, we develop the system model interested and obtain the system equations. Moreover we transform the state equations into a set of integral equations. In Section 3, numerical solutions to the integral equations are studied. In Section 4, we obtain the reliability indices of the system and their numerical simulations are presented. In Section 5, we conclude the paper. The last one the appendix, where we present how to obtain the integral equations from the system equations.

2. Description of the system model and state equations

The system we consider here consists of three-dissimilar units (called units 1, 2 and 3) and one repair facility. Initially, the system with three new units begins to operate. The system is operating if and only if unit 1 and at least one of units 2 and 3 are working. The repaired unit is as good as a new unit, and the repair discipline is "first-fail, first-repaired". Furthermore, other assumptions are given as follows:

- All random variables of the lifetime and the repair time for each unit are mutually independent.
- The failure time distributions and the repair time distributions are general: $\lambda_i(x)$ and $\mu_i(x)$ are the failure rates and repair rate functions of unit $i(i = 1, 2)$. We denote the probability density functions and distribution functions of the repair time by $f_i(t)$ and $F_i(t)$, respectively. Similarly, $g_i(t)$ and $G_i(t)$ are the probability density functions and distribution functions of the failure time, respectively. The following relations are clear:

$$F_i(t) = \int_0^t f_1(s)ds = 1 - \exp(-\int_0^t \lambda_i(s)ds) = 1 - \bar{F}_i(t), \tag{1}$$

$$G_i(t) = \int_0^t g_1(s)ds = 1 - \exp(-\int_0^t \mu_i(s)ds) = 1 - \bar{G}_i(t), \tag{2}$$

- The failure time distribution and the repair time distribution of unit 3 are exponential, and λ_3 and μ_3 are the failure rate and the repair rate, respectively.

According to these assumptions, we define the possible states of the system model as follows.

0: three units are good, units 1 and 2 are operating, and the ages of units 1 and 2 are equal;
01: three units are good, units 1 and 2 are operating, and the age of unit 1 is greater than that of unit 2;
02: three units are good, units 1 and 2 are operating, and the age of unit 1 is less than that of unit 2;
03: units 1 and 3 are operating, and unit 2 is repaired;
04: unit 3 is repaired, the units 1 and 2 are operating, and the ages of units 1 and 2 are equal;
05: unit 3 is repaired, units 1 and 2 are operating, and the age of unit 1 is greater than that of unit 2;
06: unit 3 is repaired, units 1 and 2 are operating, and the age of unit 1 is less than that of unit 2;
1: unit 1 is repaired and units 2 and 3 are shut off;
2: unit 2 is repaired, the unit 1 is waiting for repair, and unit 3 is shut off;
3: unit 2 is repaired, the unit 3 is waiting for repair, and unit 1 is operating;
4: unit 2 is repaired, the unit 3 is waiting for repair, and unit 1 is waiting for repair as well;
5: unit 3 is repaired, the unit 1 is waiting for repair, and unit 2 is shut off;
6: unit 3 is repaired, the unit 2 is waiting for repair, and unit 1 is operation;
7: unit 3 is repaired, the unit 2 is waiting for repair, and unit 1 is waiting for repair as well;
8: unit 1 is being repaired, and new unit 2 is in 'suspended animation', and unit 3 is shut off;

We define a stochastic process $S(t)$ that takes values from state space:

$$J = \{0, 01, \ldots, 06, 1, 2, \ldots, 8\}$$