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Fast simulated annealing for single-row equidistant facility layout

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ABSTRACT

Given n facilities and a flow matrix, the single-row equidistant facility layout problem (SREFLP) is to find a one-to-one assignment of n facilities to n locations equally spaced along a straight line so as to minimize the sum of the products of the flows and distances between facilities. We develop a simulated annealing (SA) algorithm for solving this problem. The algorithm provides a possibility to employ either merely pairwise interchanges of facilities or merely insertion moves or both of them. It incorporates an innovative method for computing gains of both types of moves. Experimental analysis shows that this method is significantly faster than traditional approaches. To further speed up SA, we propose a two-mode technique when for high temperatures, at each iteration, only the required gain is calculated and, for lower temperatures, the gains of all possible moves are maintained from iteration to iteration. We experimentally compare SA against the iterated tabu search (ITS) algorithm from the literature. Computational results are reported for SREFLP instances with up to 300 facilities. The results show that the performance of our SA implementation is dramatically better than that of the ITS heuristic.

1. Introduction

The *single-row equidistant facility layout problem* (SREFLP for short) is an important member of a wide set of problems whose solutions are permutations. It can be stated as follows. Suppose that there are n facilities and, in addition, there is an $n \times n$ symmetric matrix, $W = (w_{ij})$, whose entry w_{ij} represents the flow of material between facilities i and j. The SREFLP is to find a one-to-one assignment of n facilities to n locations equally spaced along a straight line so as to minimize the sum of the products of the flows and distances between facilities. It can be assumed without loss of generality that the locations are points on a horizontal line with x-coordinates $1, 2, \ldots, n$. Then, mathematically, the SREFLP can be expressed as:

$$\min_{p \in \Pi} F(p) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} w_{p(i)p(j)}(j - i), \quad (1)$$

where Π is the set of all permutations of $I = \{1, \ldots, n\}$, which is defined as the set of all vectors $(p(1), p(2), \ldots, p(n))$ such that $p(i) \in I, i = 1, \ldots, n$, and $p(i) \neq p(j), i = 1, \ldots, n-1, j = i+1, \ldots, n$. Thus $p(i), i \in I$, is the facility assigned to location i.

The applicability of the SREFLP has been reported in several settings. Yu and Sarker [44] considered this problem in the context of designing a cellular manufacturing system. In this application, the goal is to assign machine-cells to locations so that the total

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inter-cell flow costs would be minimized. An important condition is that the locations must be equally spaced in a linear layout. Sarker et al. [37] investigated a similar problem arising in flexible manufacturing. Its objective is to minimize the total machine-to-machine material transportation cost when machines are assigned to locations along a linear material handling track. Picard and Queyranne [32] addressed the problem of designing the physical arrangement of rooms within a department. In this case, the matrix W represents the traffic, i.e. its entry w_{ij} equals the number of trips per unit of time between room i and room j. The quality of a layout is measured by the total travel distance, which is defined as the sum of the door-to-door distances weighted by the traffic between the rooms. When all room lengths are equal, the problem can be modeled as the SREFLP. Cheng [11] and Bhasker and Sahni [8] applied the model (1) in the area of physical design automation of electronic systems. They considered a linear placement problem, which asks to arrange circuit components on a straight line so as to minimize the objective function of the sum of wiring lengths. Other applications of the SREFLP are found in sheet metal fabrication [4], printed circuit board and disk drive assembly [12], and assigning flights to gates in an airport terminal [40].

The SREFLP, as defined by (1), is a special case of the quadratic assignment problem (QAP) formulated by Koopmans and Beckmann [23]. In recent years, a number of high-performing algorithms for the QAP have been proposed, including those presented in [1,6,7,14,15,20,21,39,42]. For a survey of less recent algorithms for the QAP, the reader is referred to Loiola et al. [27]. A good account of results on the QAP can also be found in the monograph by Burkard et al. [9]. However, the algorithms for the SREFLP generally differ from those for the QAP, because they take advantage of the fact that the locations for facilities are equally spaced along a straight line. Therefore, the development of specific algorithms for the SREFLP is an important line of research. There are a few more combinatorial optimization problems related to the SREFLP. One of them is the minimum linear arrangement (MinLA) problem, which asks to find a labeling of the vertices of a given undirected n-vertex graph by consecutive integers from 1 to n such that the sum of edge lengths is minimized, where the length of an edge is the absolute difference of the labels on its end vertices. The graphs arising in applications of the MinLA, for example, in graph drawing tend to be rather sparse. The most successful heuristic techniques for the MinLA problem include the multilevel algorithm based on the algebraic multigrid scheme by Safro et al. [35] and the two-stage simulated annealing algorithm by Rodriguez-Tello et al. [33]. An overview of results in the area of MinLA is presented in a survey by Díaz et al. [13]. Another related problem is the single-row facility layout problem (SRFLP) in which the facilities, represented by rectangles, are allowed to have different lengths. Given the matrix of flows, the problem is to find an arrangement of the facilities next to each other along a horizontal line such that the weighted sum of the center-to-center distances between all pairs of facilities is minimized. Unlike the SREFLP, this problem is not a special case of the QAP. The state of the art on single-row facility layout is surveyed in the papers by Kothari and Ghosh [24] and Hungerländer and Rendl [18]. A natural generalization of the SRFLP is the corridor allocation problem (CAP) considered in [2,3]. This problem asks to find an arrangement of the facilities on both sides of the corridor leaving no space between two adjacent facilities. The objective function of the CAP is essentially the same as that of the SRFLP.

A number of approaches, both exact and heuristic, for the SREFLP have been proposed in the literature. Karp and Held [22] developed an exact algorithm based on the dynamic programming technique. The time complexity of this algorithm is $O(n2^n)$ [22,32]. However, the algorithm can only handle problem instances of moderate size because it requires a very large memory space to store the partial solutions [26]. A branch-and-bound algorithm for solving the SREFLP was presented in [30]. Using it, a number of benchmark instances with up to 35 facilities were solved to optimality. Recently, Hungerländer [17] proposed a semidefinite programming approach for the SREFLP. The author developed an algorithm that was able to solve instances of size up to 40.

It is well known that the SREFLP given by (1) is NP-hard in its general form. Thus, SREFLP instances of larger size can be tackled only using heuristic optimization methods. Suryanarayanan et al. [40] developed a heuristic algorithm based on the multidimensional scaling technique. The solution obtained by applying this technique was further improved using a pairwise interchange method. The authors reported computational results for problem instances of size up to 40 facilities. Sarker et al. [37] proposed an algorithm called the depth-first insertion heuristic (DIH). This heuristic starts with a feasible solution (permutation of facilities) and iteratively tries to improve it. At each iteration, DIH examines all assignments obtained by removing the selected facility from its current position in the permutation and inserting it at each of the other $n-1$ positions. While performing this operation, some of the other facilities are shifted to the neighboring locations. As remarked in [37], the time complexity of DIH is $O(n^4)$. Yu and Sarker [44] presented another algorithm for the SREFLP, called directional decomposition heuristic (DDH). This algorithm bears some similarity to DIH but is computationally more efficient. In [30], an iterated tabu search (ITS) algorithm for solving the SREFLP was proposed. A tabu search procedure lies at the core of a dominance test incorporated into a branch-and-bound algorithm described in [30]. As the results of numerical experiments provided in that paper show, ITS performs considerably better than the DDH method.

The primary motivation of this paper is to investigate computationally the applicability of the simulated annealing (SA) metaheuristic to the single-row equidistant facility layout problem. We consider two move types, pairwise interchanges of facilities and insertions (when a facility is removed from its current position and inserted at a different position in the permutation). We propose an original method to compute the move gains, that is, the differences between the objective function values of the new and current solutions. Interestingly, this method operates on the same auxiliary data for both move types. Another innovative aspect of the work presented in this paper is the use of the two modes of calculating move gains. For high temperatures, we adopt the traditional approach when, at each iteration of the SA algorithm, first a pair of facilities (respectively, a facility and its new position) are selected and then the gain is computed in $O(n)$ (respectively, $O(n^2)$) operations. For lower temperatures, our strategy is to maintain, throughout the cooling process, special matrices and vectors that allow computing the gain of any pairwise interchange or insertion move in $O(1)$ time. The speedup provided by this approach is significant. We have implemented